

EXAM 3
Math 106
November 20, 2002

Name _____

You must show all your work. Partial credit will be given.

1. Three hundred dollars invested in an account that pays 6.5% interest, compounded monthly, will generate an amount $A = 300(1.0054^{12t})$ dollars in t years.

(a) How much is in the account after two years and what is the rate of change of the amount in the account at two years? (8 pts)

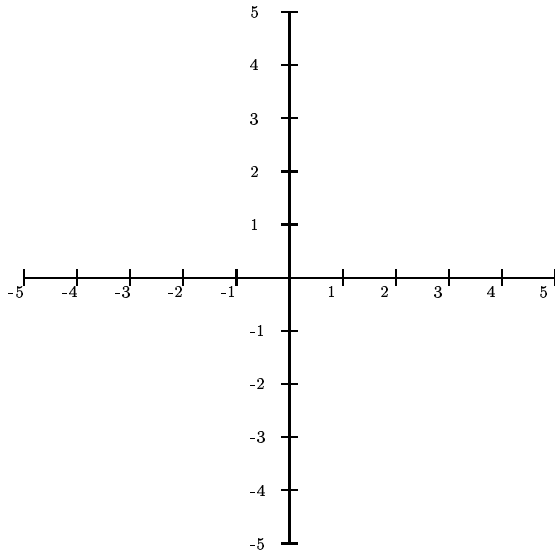
(b) Using part *a* estimate the amount you would expect to be in the account after three years. (Do not use the equation to find an exact amount, use your previous answers to arrive at an estimate.) (5 pts)

2. Give **estimates** for each of the following

(a) If an airplane is flying 300 mph and is accelerating at a rate of 200 mph per hour, estimate the airplane's speed in 20 minutes. (5 pts)

(b) If $g(7) = 4$ and $g'(7) = -12.8$ estimate $g(7.5)$. (5 pts)

3. Sketch the graph of a function f such that
 $f'(x) > 0$ for $x < -1$ and $x > 3$
 $f'(x) < 0$ for $-1 < x < 3$
 $f'(-1) = 0$ and $f'(3)$ does not exist. (9 pts)



4. The flow rate (in cubic feet per second, cfs) of a river in the 24 hours after the beginning of a severe thunderstorm can be modeled by

$$C(h) = \begin{cases} 12.045h^2 - 8.952h + 123.02 & \text{cfs when } 0 \leq h \leq 10 \\ -16.643h + 539.429 & \text{cfs when } 10 \leq h \leq 24 \end{cases}$$

- (a) What were the flow rates for $h = 0$ and $h = 24$ respectively? (6 pts)

- (b) Determine the absolute maximum and minimum flow rates between $h = 0$ and $h = 24$. (10 pts)

5. Accelerations (rate of change in speed) for a vehicle during a road test are approximated in the following table.

| Time (seconds) | Accelerations (feet per second squared) |
|----------------|---|
| 0 | 22.6 |
| 2 | 18.2 |
| 4 | 14.5 |
| 6 | 11.4 |
| 8 | 8.9 |
| 10 | 7.1 |
| 12 | 5.9 |

(a) Find an exponential model for this data. (5 pts)

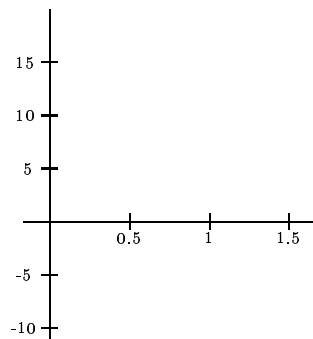
(b) Use 4 left-side rectangles to estimate the area of the region between the graph of your model and the input axis from 0 to 13.5 seconds. Interpret your answer. (15 pts)

6. The rate of change of the temperature during the hour and a half after a thunderstorm began is modeled by the equation

$$T(h) = 9.48h^3 - 15.49h^2 + 17.38h - 9.87$$

degrees Fahrenheit per hour where h is the number of hours since the storm began.

(a) Graph the equation on the axis below and determine where the function crosses the horizontal axis. (The h -axis.) (5 pts)

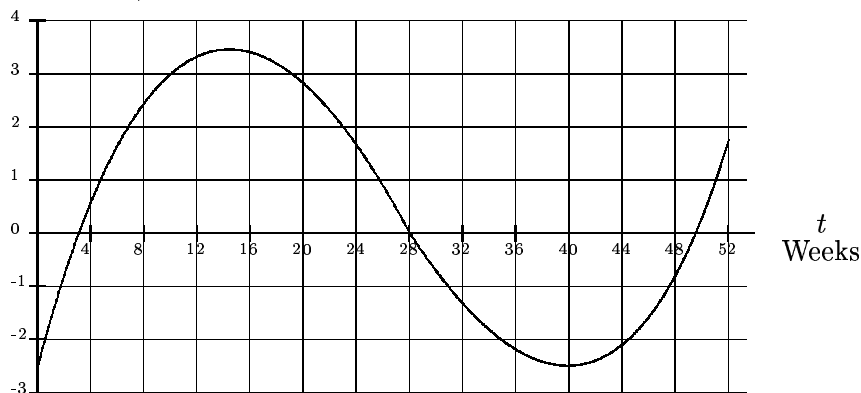


(b) Use the concept of a limit of sums to **estimate** the value of $\int_A^{1.5} T(h) dh$, where A is the value you found in part *a*. (That is, the place where the graph crosses the axis.) (10 pts)

(c) If the temperature was 73° at time $h = A$ what is the temperature at time $h = 1.5$? (5 pts)

7. The following graph is a model of the rate of change of profit for a new business during its first year. The input is the number of weeks since the business opened, and the output units are thousands of dollars per week.

$p(t)$
(thousands of
dollars per week)



(a) What does the area of each box in the grid represent? (5 pts)

(b) Count boxes to estimate the accumulation values $(\int_0^x p(t) dt)$ from 0 to x for the values given in the table, then graph the accumulation function on the axis given. (10 pts)

| x | Accumulation function value | x | Accumulation function value |
|-----|-----------------------------|-----|-----------------------------|
| 0 | | 28 | |
| 4 | | 32 | |
| 8 | | 36 | |
| 12 | | 40 | |
| 16 | | 44 | |
| 20 | | 48 | |
| 24 | | 52 | |

