Final

Math 115

Dec. 13, 2000

Name ______________________________

1. Find each of the following. (7 pts each)

   (a) $\displaystyle \lim_{x\to -2} \frac{x^2 - x - 6}{x + 2}$.

   (b) $\displaystyle \lim_{x\to \infty} \frac{2x^2 - 1}{3x^4 + 2}$.

2. Find the derivative of each of the following. (7 pts each)

   (a) $y = (x^3 + 2)(x^2 - 1)^3$.
   $y' =$

(b) $f(x) = \dfrac{x^2+1}{e^{3x}-2}$.
$f'(x) =$

(c) $f(x) = \sqrt{\ln x^2 + 1}$.
$f'(x) =$

3. Let $f(x) = 4x^2 + \dfrac{1}{x}$. Find the intervals on which f is increasing/decreasing and any relative maximum and/or minimum values. (9 pts)

4. Let $y^3 + xy^2 + 1 = x + 2y^2$. Find $\frac{dy}{dx}$. (8 pts)

5. Find the equation of the tangent line to the curve $y + \frac{\sqrt{x}}{y} = 3$ at the point $(4, 2)$. (8 pts)

6. Let $f(x) = \dfrac{x}{2x+1}$. Find $f^{(3)}(x)$. (9 pts)

7. Let $f(x) = -x^2e^x$. Then $f'(x) = e^x(-x^2 - 2x)$ and $f''(x) = e^x(-x^2 - 4x - 2)$. Find the intervals on which f is increasing/decreasing, concave up/down any relative maximum or minimum values and any points of inflection. (9 pts)

8. Let $f(x) = \dfrac{x-1}{x^2+1}$. Find the absolute maximum and the absolute minimum of f on the interval $[1, 5]$. (7 pts)

9. Sociologists have found that crime rates are influenced by temperature. In a midwestern town of 100,000 people, the crime rate has been approximated as

$$C = \frac{1}{10}(T-60)^2 + 100,$$

where C is the number of crimes per month and T is the monthly temperature. The average temperature for May was 76°, and by the end of May the temperature was rising at the rate of 8° per month. How fast is the crime rate rising at the end of May? (8 pts)

10. Find each of the following. (7 pts each)

(a) $\displaystyle\int z\sqrt{z^2-5}\,dz.$

(b) $\displaystyle\int (x^2-1)e^{x^3-3x}\,dx.$

(c) $\displaystyle\int_0^1 2(t^{\frac{1}{2}}-t)\,dt.$

(d) $\displaystyle\int_1^2 \frac{1}{x(1+\ln x)}\,dx.$

11. Find the area between the two curves $y = x^2$ and $y = x^3$. (10 pts)

12. Let $f(x, y) = y^2 e^{x+3y}$. Find $f_x(x, y)$ and $f_y(x, y)$. What is $f_x(-3, 1)$? (12 pts)

13. Let $f(x, y, z) = \ln |x^2 - 5xz^2 + y^4|$. Find $f_{yz}(x, y, z)$. (8 pts)