

Lab 9

Name _____

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The purpose of lab 8 is to give you practice in utilizing derivatives when combined with families of functions.

1. The study of resonance leads to the family of functions

$$y = \frac{1}{(1-x^2)^2 + 2ax^2}, \quad x \geq 0, \quad a > 0.$$

- (a) Find and classify the critical points, and then explain why the family is most interesting for $0 < a < 1$.
 - (b) Show that for a very near 0, there is a critical point at approximately $(1, \frac{1}{2a})$.
 - (c) Show that $y < \frac{1}{(1-x^2)^2}$ for $x > 0$.
 - (d) Sketch the curves for $a = 0.05, 0.10, 1.0,$ and 3.0 on one set of axes together with $y = \frac{1}{(1-x^2)^2}$, which is the curve with $a = 0$.
 - (e) What is the geometric significance of the parameter a ?
2. Your parents are going to knock out the bottom of the entire length, l , of the south wall of their house and turn it into a greenhouse by replacing the bottom portion of the wall with a huge sloped peice of glass (which is expensive). They have already decided they are going to spend a certain fixed amount, k . The triangular ends of the greenhouse will be made of various materials they already have lying around.

The floor space in the greenhouse is only considered usable if they can both stand up in it, so part of it will be unusable. They want to choose the dimensions of the greenhouse to get the most usable floor space. What should the dimensions of the greenhouse be and how much usable space will your parents get? (You should end up with an expression in $l, k,$ and h , where h is the height of the tallest parent.)

(hints:

- (a) Let x be the distance the tallest parent can walk into the greenhouse then get x as an expression of width of the glass and the angle the glass makes with the ground.
- (b) Find the critical points.
- (c) Maximize the area.)