EXAM 3

Math 132

January 15, 2009

Name _____

1. Find the nth term for the sequence of numbers, $\frac{1}{3}, \frac{2}{5}, \frac{3}{7}, \frac{4}{9}, \frac{5}{11}, \ldots$. Does this sequence converge to a number? Explain. (8 pts)

2. Is the series $2 - \frac{2}{3} + \frac{2}{9} - \frac{2}{27} + \frac{2}{81} - \ldots$ a geometric series? If not explain why not, if so find the sum of the series. (8 pts)

3. For each of the following series decide whether it converges or diverges and explain your decision. If it converges and if it is possible find the sum. (10 pts each)

   (a) $\displaystyle\sum_{n=1}^{\infty} \frac{\sqrt{n}}{(n^{3/2}+3)}$

(b) $\displaystyle\sum_{n=1}^{\infty} \frac{3^n}{2+n^2}$

(c) $\displaystyle\sum_{n=0}^{\infty} \frac{3^n}{(n+2)!}$

(d) $\sum_{n=2}^{\infty} \frac{n}{\sqrt{n^5-6}}$

4. Find the radius of convergence of (6 pts)

$2(x+5)+3(x+5)^2+\frac{4(x+5)^3}{2!}+\frac{5(x+5)^4}{3!}+\frac{6(x+5)^5}{4!}+\dots$. (8 pts)

5. Find the 4th degree Taylor Polynomial, centered at 1, for the function $f(x) = \ln(x)$. Use your result to estimate the value of $\ln(2)$. (You must use your result, you may use your calculator as an aid but simply plugging in $\ln(2)$ does not count.) (12 pts)

6. Find a Taylor Series expansion for the function $f(t) = t\sin(3t)$. State where you chose to center your series (i.e. what is a) and write the nth term. Show your reasoning. (12 pts)

7. The Taylor Series expansion for the function $f(x) = \ln(1+x)$ (expanded about 0) is $T(x) = x - x^2/2 + x^3/3 - x^4/4 + x^5/5 - \cdots = \sum_{n=1}^{\infty} \frac{(-1)^{n-1}x^n}{n}$. Use this fact to estimate $\ln(1.1)$ with a 4th degree polynomial and then find a bound for the error of that estimate. (12 pts)

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