EXAM 2
Math 232
August 3, 2007

Name ______________________________

1. Show that $\displaystyle\lim_{(x,y)\to(0,0)} \frac{x^2yz}{x^4+y^4+z^4}$ does not exist. (6 pts)

2. Find the tangent plane of $f(x,y) = \sin(x)\cos(y)$ at the point $(0,\pi)$. (6 pts)

3. Find the linear approximation of $f(x,y) = \sqrt{x^2+y^2}$ at the point $(3,0)$ then use this approximation to estimate the value of the function at $(3,1)$. (6 pts)

4. Find each of the following. (6 pts each)

(a) f_x if $f(x,y) = 3x^4y + 8x^{0.5}y^2 - 3x^2$

(b) $\dfrac{\partial}{\partial h}\left(\dfrac{13}{2a}(2a+b)h\right)$

(c) $\dfrac{\partial^2 z}{\partial y \partial x}$ if $z = e^{x+2y}\sin y$.

(d) $f_x(\pi/3, 1)$ if $f(x,y) = x\ln\left(y\cos\left(x\right)\right)$

5. Find the absolute extrema for the function $f(x, y) = x^2 + y^2 - 2x - 4y$ on the region bounded by $y = x$, $y = 3$ and $x = 0$. (6 pts)

6. Integrate $\displaystyle\int_R\int e^{x^2}\, dA$ where R is bounded by $x = 4 - y^2$ and $x = 4$. (6 pts)

7. Find $\displaystyle\int_0^1 \int_{\sqrt{x}}^1 \frac{3}{4 + y^3}\, dy\, dx$ (6 pts)

8. Find the volume of the solid bounded by $z = x^2 + y^2 + 3$, $z = -2$, $y = x^2$ and $y = 4$. (You may use double or triple integrals as you prefer.) (8 pts)

9. Find the mass and the center of mass of the lamina bounded by $x = y^2$ and $x = 1$ with a density of $\rho(x, y) = y^2 + x + 1$. (6 pts)

10. Find $\displaystyle \int_0^2 \int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}} e^{-x^2-y^2}\, dy\, dx$. (Hint: polar coordinates) (6 pts)

11. Integrate (triple integral) the function $f(x, y, z) = 3y^2 - 2z$ over Q where Q is the region bounded by the plane $3x + 2y - z = 6$, the xy-plane, the xz-plane and the yz-plane. (This is a tetrahedron.) (6 pts)

12. Find $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$ for the formula $z^2 = x^2 - y^2$. Now use these formulas to write the equation of the plane tangent to the surface at $5, -3, -4)$. (6 pts)

13. Find both $\dfrac{\partial f}{\partial u}$ and $\dfrac{\partial f}{\partial v}$ for the following function $z = \tan^{-1}\left(\dfrac{x}{y}\right)$ with $x = u^2 + v^2$ and $y = u^2 - v^2$. (6 pts)

14. Find the directional derivative of $f(x, y) = x^2 \sin(4y)$ at the point $\left(-2, \dfrac{\pi}{8}\right)$ in the direction $\overrightarrow{u} = \langle 2, -1 \rangle$. (6 pts)

15. Find the direction of the maximum change in $f(x, y) = y^2 e^{4x}$ at the point $(3, -1)$. (6 pts)

16. Find and classify all critical points for the function $f(x, y) = x^2 e^{-x^2-y^2}$. (8 pts)