EXAM 2

Math 232

August 1, 2008

Name ______________________________

1. Find the tangent plane of $f(x, y) = x^3 - 2xy$ at the point $(-2, 3)$. (6 pts)

2. Find the linear approximation of $f(x, y) = \sqrt{x^2 + y^2}$ at the point $(3, 0)$ then use this approximation to estimate the value of the function at $(3, 1)$. (6 pts)

3. Find each of the following. (5 pts each)

(a) f_x if $f(x, y) = x^4 - ex^2y^3 + 5y$

(b) $f_{yy}(\pi/2, 2)$ if $f(x, y) = 3x \sin(y) + 4x^3y^2$

(c) $\dfrac{\partial}{\partial y}\sqrt{x^2 + y^2}$

(d) $\dfrac{\partial^2 z}{\partial y \partial x}$ if $z = e^{x+2y} \sin y$.

4. Find both $\frac{\partial f}{\partial u}$ and $\frac{\partial f}{\partial v}$ for the following function $z = 4x^2y^3$ with $x = u^3 + v\sin(u)$ and $y = 4u^2$. (6 pts)

5. Find the directional derivative of $f(x, y) = x^2 \sin(4y)$ at the point $\left(-2, \frac{\pi}{8}\right)$ in the direction $\vec{u} = \langle 2, -1 \rangle$. (5 pts)

6. Find the absolute extrema for the function $f(x, y) = x^2 + 3y - 3xy$ on the region bounded by $y = x$, $y = 0$ and $x = 2$. (6 pts)

7. Integrate $\displaystyle\int_R\int 2e^{y^2}\, dA$ where R is bounded by $y = x$ and $y = 2$ and $x = 0$. (6 pts)

8. Find $\displaystyle\int_0^1\int_0^{y^2} \frac{3}{4+y^3}\, dx\, dy$ (6 pts)

9. Find the volume of the solid bounded by $z = 3x^2 + 2y$, $z = 0$, $y = 1 - x^2$ and $y = 0$. (You may use double or triple integrals as you prefer.) (6 pts)

10. Find the mass and the center of mass of the lamina bounded by $x = y^2$ and $x = 4$ with a density of $\rho(x, y) = y + 3$. (6 pts)

11. Find $\displaystyle\int_0^2 \int_x^{\sqrt{8-x^2}} (-x^2 - y^2)^{\frac{3}{2}}\, dy\, dx$. (Hint: polar coordinates) (5 pts)

12. Integrate (triple integral) the function $f(x, y, z) = 3y^2 - 2z$ over Q where Q is the region bounded by the plane $3x + 2y - z = 6$, the xy-plane, the xz-plane and the yz-plane. (This is a tetrahedron.) (6 pts)

13. Find $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$ for the formula $3yz^2 - e^{4x}\cos(4z) - 3y^2 = 4$. (6 pts)

14. Find the direction of the maximum change in $f(x, y) = y^2 e^{4x}$ at the point $(3, -1)$. (5 pts)

15. Find and classify all critical points for the function $f(x, y) = x \sin(y)$. (6 pts)