

**EXAM 2**  
Math 232  
April 10, 2008

Name \_\_\_\_\_

1. Let  $x, y \in G$  with  $x$  and  $y$  both not the identity. If  $G$  is a group of order 161 show that there is only one subgroup of  $G$  which contains both  $x$  and  $y$ , namely  $G$  itself.

2. Decide whether the order of  $U(232)$  is odd or even. Do the same for  $U(154)$ .

3. Suppose that two pairs of groups are isomorphic, i.e.  $G \cong G'$  and  $H \cong H'$ . Show that  $G \oplus H \cong G' \oplus H'$ .

4. Determine  $\text{Aut}(Z_2 \oplus Z_2)$

5. Find an isomorphism from  $Z_{12}$  to  $Z_4 \oplus Z_3$ .

6. Express  $U(165)$  as an external direct product of U-groups in four different ways

7. In  $Z$  let  $H = \langle 5 \rangle$  and  $K = \langle 7 \rangle$ . Prove that  $Z = HK$ . Does  $Z = H \times K$ ?

8. If  $|G| = pq$  where  $p$  and  $q$  are primes that are not necessarily distinct, prove that  $|Z(G)| = 1$  or  $pq$ .

9. How many homomorphisms are there from  $Z_{20}$  onto  $Z_8$ ? How many are there if the onto requirement is dropped?

10. Suppose there is a homomorphism from a finite group  $G$  onto  $Z_{10}$ . Prove that  $G$  has a normal subgroups of indexes 2 and 5.