Maple Worksheets for Ordinary Differential Equations

Complimentary software to accompany the textbook:
Differential Equations: Concepts, Methods, and Models (2001-2002 Edition)
Leigh C. Becker
Department of Mathematics
Christian Brothers University 650 East Parkway South Memphis, TN 38104
Telephone: (901) 321-3443 FAX: (901) 321-3494 E-mail: lbecker@cbu.edu http://www.cbu.edu/~lbecker

Examples of the Use of the Dirac Delta Function

>
> **restart:**

The Slot Machine Example

>
> **Net_Worth := 100 + 45*Heaviside(t-1);**

$$Net\_Worth := 100 + 45\ \text{Heaviside}(t-1)$$

>
Change the net worth expression into a function. Call it *x*(*t*), the net worth at time *t*:
> **x :=unapply(Net_Worth,t);**

$$x := t \rightarrow 100 + 45\ \text{Heaviside}(t-1)$$

>
Rate of change in the net worth is the derivative of *x*(*t*) with respect to the time *t*:
> **diff(x(t),t);**

$$45\ \text{Dirac}(t-1)$$

> **x(t);**

$$100 + 45\ \text{Heaviside}(t-1)$$

>
> **x(0);**

$$100$$

>
> **x(2);**

$$145$$

>
Reset *x*:
>
> **x := 'x';**

$$x := x$$

> **x(2);**

$$\text{x}(2)$$

Now pretend the net worth function is unknown but that we do know the rate of change in the net worth:
>
> **Rate := diff(x(t),t) = 45*Dirac(t-1);**

$$Rate := \frac{d}{dt}\text{x}(t) = 45\ \text{Dirac}(t-1)$$

>
Initial Condition:

```
> IC := x(0) = 100;
```

$$IC := x(0) = 100$$

```
>
```

Now solve the intial value problem for the net worth:

```
>
> Soln := dsolve({Rate, IC}, x(t));
```

$$Soln := x(t) = 100 + 45\ \text{Heaviside}(t - 1)$$

```
>
> N := unapply(Soln, t);
```

$$N := t \to x(t) = 100 + 45\ \text{Heaviside}(t - 1)$$

```
>
> N(0);
```

$$x(0) = 100$$

```
>
> N(2);
```

$$x(2) = 145$$

```
>
```

Definition of the Dirac delta function

```
>
> Heaviside(t - T);
```

$$\text{Heaviside}(t - T)$$

```
> diff(Heaviside(t-T), t);
```

$$\text{Dirac}(t - T)$$

```
>
```

Graphs of the Heaviside function and the Dirac delta function

```
>
```

Let's assign *T* an arbitrary value, such as T = 2:

```
> T := 2;
```

$$T := 2$$

```
> Heaviside(3.5 - T);
```

$$1.$$

```
> Dirac(3.5 - T);
```

$$0$$

```
>
> Heaviside(1.8 - T);
```

$$0.$$

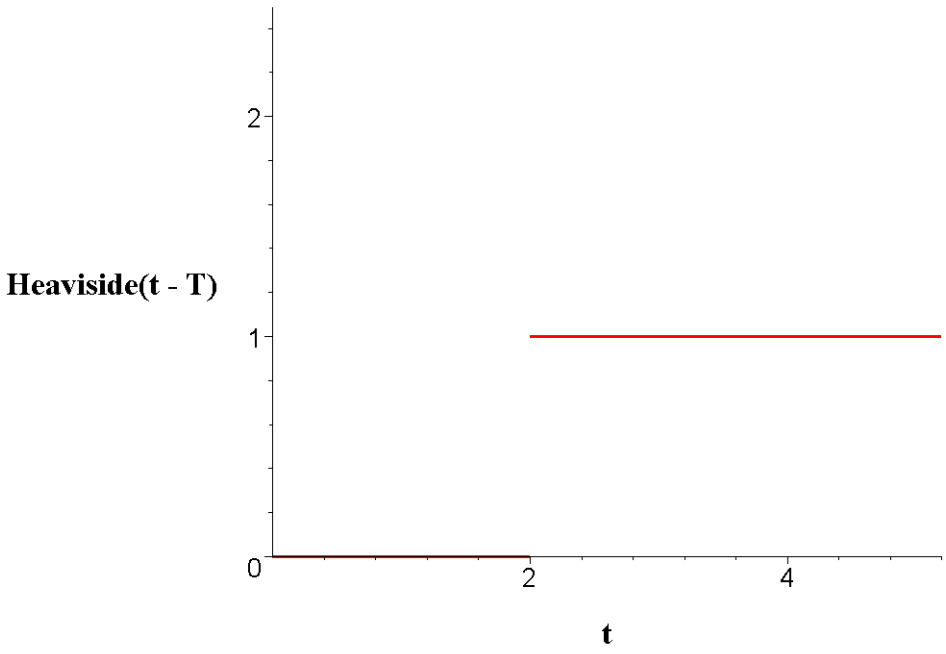
```
> Dirac(1.8 - T);
```

$$0$$

```
>
> convert(Heaviside(t -T), piecewise, t);
```

$$\begin{cases} 0 & t < 2 \\ \mathit{undefined} & t = 2 \\ 1 & 2 < t \end{cases}$$

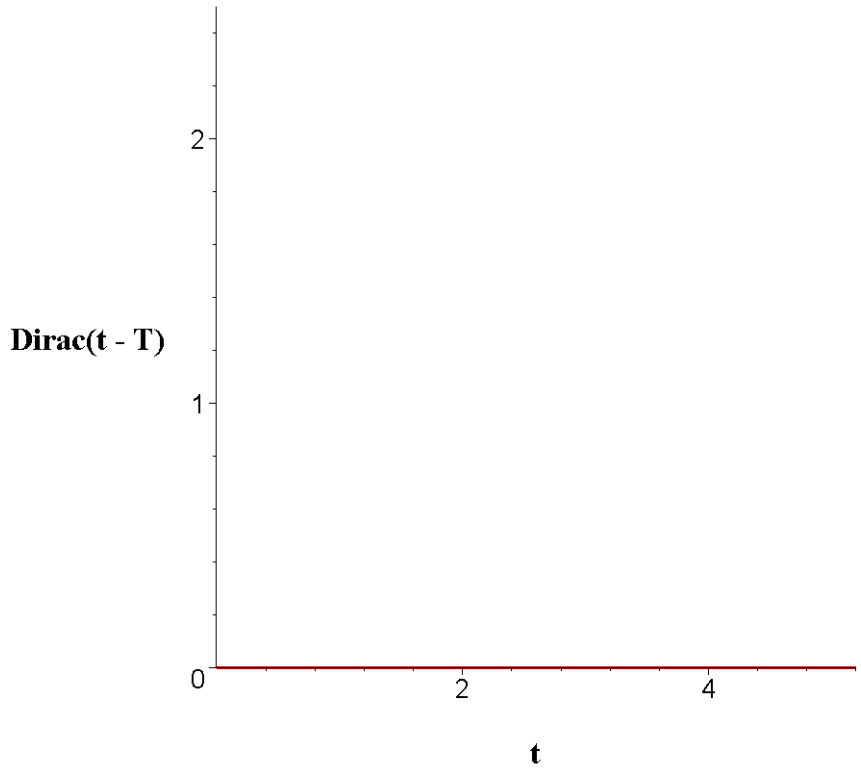
```
>
> plot (Heaviside(t -T), t = 0..5.2, H = 0..2.5, tickmarks = [3, 3], labels = [t,"Heaviside(t - T)"], discont=true,
  thickness=3,  color = red, labelfont = [TIMES,BOLD,12]);
```




```
> 
> convert(Dirac(t-2), piecewise, t);
```

$$\begin{cases} \mathit{undefined} & t = 2 \\ 0 & \mathit{otherwise} \end{cases}$$

```
> 
> plot (Dirac(t-T), t= 0..5.2, H = 0..2.5, tickmarks = [3,3], labels = [t,"Dirac(t - T)"],
  discont=true, thickness=3, color = red, labelfont = [TIMES,BOLD,12]);
```



>

>

Integration of the Dirac delta function

>

> **Int(Dirac(t - T), t = 0 .. 4);**

$$\int_0^4 \mathrm{Dirac}(t-2)\,dt$$

>

> **Int(Dirac(t -T), t = 0 .. 4) = int(Dirac(t - T), t = 0..T);**

$$\int_0^4 \mathrm{Dirac}(t-2)\,dt = 1$$

>

>

An Impulse Function

>

A Dirac function can be used to model an *impulse function*.

> **tau := 0.05; T := 2;**

$$\tau := 0.05$$

$$T := 2$$

>

> **Impulse := (1/tau)*(Heaviside(t - T) - Heaviside(t -T - tau));**

$$Impulse := 20.00000000\ \text{Heaviside}(t - 2) - 20.00000000\ \text{Heaviside}(t - 2.05)$$

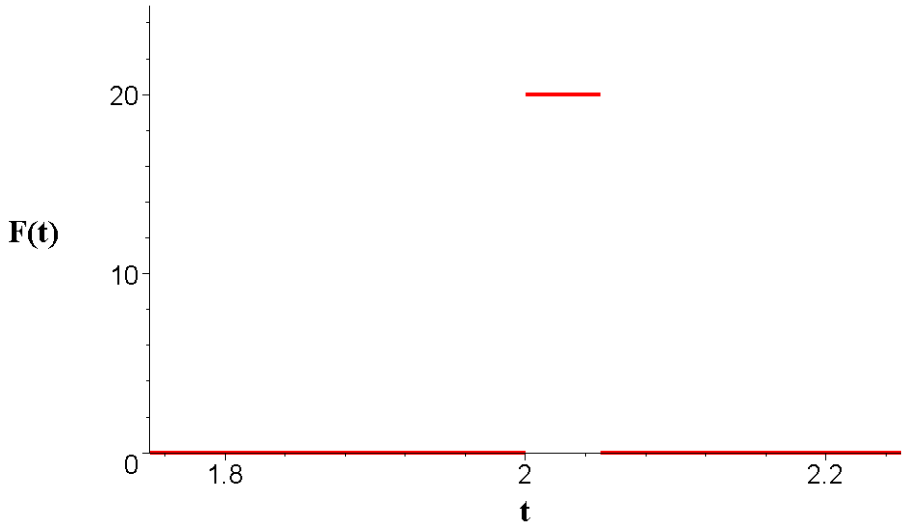
>

```
> F := unapply(Impulse, t);
```

$$F := t \rightarrow 20.00000000\ \text{Heaviside}(t - 2) - 20.00000000\ \text{Heaviside}(t - 2.05)$$

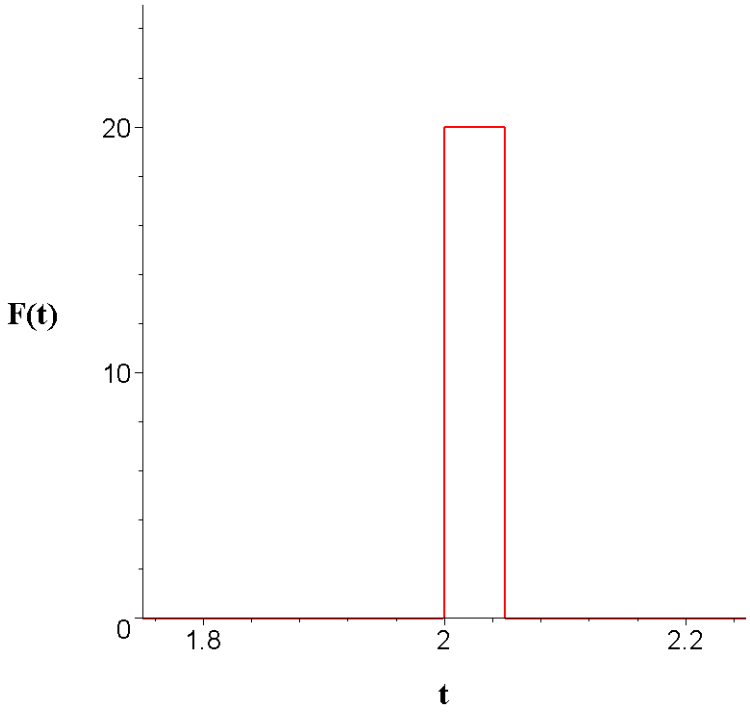
>

```
> plot (F(t), t = 2-5*tau..2+5*tau,  F = 0..5+1/tau, tickmarks = [3,3], labels = [t,"F(t)   "],
  discont = true, thickness = 4, color = red, labelfont = [TIMES,BOLD,12]);
```



>

```
> plot (F(t), t = 2-5*tau..2+5*tau,  F = 0..5+1/tau, tickmarks = [3,3], labels = [t,"F(t)   "],
  thickness = 2, color = red, labelfont = [TIMES,BOLD,12]);
```



>

>

```
> convert(F(t), piecewise, t);
```

$$\begin{cases} 0. & t < 2 \\ \textit{undefined} & t = 2 \\ 20.00000000 & t < 2.05 \\ \textit{undefined} & t = 2.05 \\ 0. & 2.05 < t \end{cases}$$

>

> **Int(F(t), t = 0..4) = int(F(t), t = 0..4);**

$$\int_0^4 20.00000000\ \text{Heaviside}(t-2) - 20.00000000\ \text{Heaviside}(t-2.05)\,dt = 1.$$

>

**** END ****

>

>

>