MATH 105 – Exam 1 (Practice) **Name** Key

September 19, 2008

This is a sample test. **Caveat.** This sample test represents some of the kinds of problems that we have covered. However, you are responsible for all the kinds (both "practice" and "turn-in") of homework problems that I have assigned and also the reading assignments.

Part 1. Calculators are not allowed on this part. Show all of your work. Circle all answers.

1. Find the equation of the line that passes through the points $(-2,10)$ and $(4,34)$. (10 pts)

First find the slope m: $m = \frac{\Delta y}{\Delta x} = \frac{34-10}{4-(-2)} = \frac{24}{6} = 4$

By the point-slope form of a line, we have

$y - y_1 = m(x - x_1) \Rightarrow y - 10 = 4(x-(-2)) \Rightarrow y - 10 = 4(x+2)$

$\Rightarrow y = 4x + 8 + 10 \Rightarrow \boxed{y = 4x + 18}$

2. **a)** Find the domain of $f(x) = \dfrac{-7x+3}{5x+4}$.

f is not defined where $5x+4=0$.

Solving, $x = -\frac{4}{5}$. Therefore,

the domain is all real numbers except for $x = -\frac{4}{5}$.

(b) Evaluate $f(-1)$

$f(-1) = \dfrac{-7(-1)+3}{5(-1)+4} = \dfrac{10}{-1} = \boxed{-10}$

(c) Find the domain of $g(x) = \dfrac{\sqrt{2x-3}}{x-5}$.

g is not defined where $x-5=0$. So $x=5$ is not in the domain of g. Also, $2x-3$ must be nonnegative: $2x-5 \geq 0$. So, $x \geq \frac{5}{2}$.

Therefore the domain of g is all real nos greater than or equal to 2.5 except for $x=5$.

(d) Evaluate $g(2)$

$g(2) = \dfrac{\sqrt{2(2)-3}}{2-5} = \dfrac{\sqrt{1}}{-3} = -\frac{1}{3}$

3. Office equipment was purchased for \$30,000 and is assumed to have a scrap value of \$4,000 after 10 years. (10 pts)

(a) If its value V is depreciated linearly (for tax purposes) from \$30,000 to \$4,000, find a linear model that relates the value in dollars of the equipment to the time t in years.

Since V depreciates linearly, it has the form $V = mt + b$.

At $t=0$, $V = \$30000$ and $V = m(0) + b = b$. Thus, $b = 30{,}000$.

The slope is $m = \dfrac{\Delta V}{\Delta t} = \dfrac{4000-30000}{10-0} = \dfrac{-26000}{10} = -2600$.

Therefore, $V = -2600t + 30{,}000$.

(b) What is the value of the equipment after 5 years?

when $t = 5$,

$V = -2600(5) + 30000 = \$17{,}000$

4. The Slower Than Slow Freight Company charges \$5.00 per pound to ship freight with a minimum cost of \$40.00. Write a piecewise linear function to model the cost of shipping x pounds of freight. (? pts)

Let $C(x)$ be the cost of shipping x pounds of freight.

$5x = 40 \Rightarrow x = \frac{40}{5} = 8$ lbs.

Thus, $C(x) = \begin{cases} \$40, & \text{if } x \le 8 \text{ lbs} \\ 5x \text{ dollars}, & \text{if } x > 8 \text{ lbs} \end{cases}$.

5. Find the x-intercept and y-intercept of $3y - 8x = 15$ and graph it. (? pts)

When $x=0$, we have

$3y - 8(0) = 15 \Rightarrow 3y = 15$

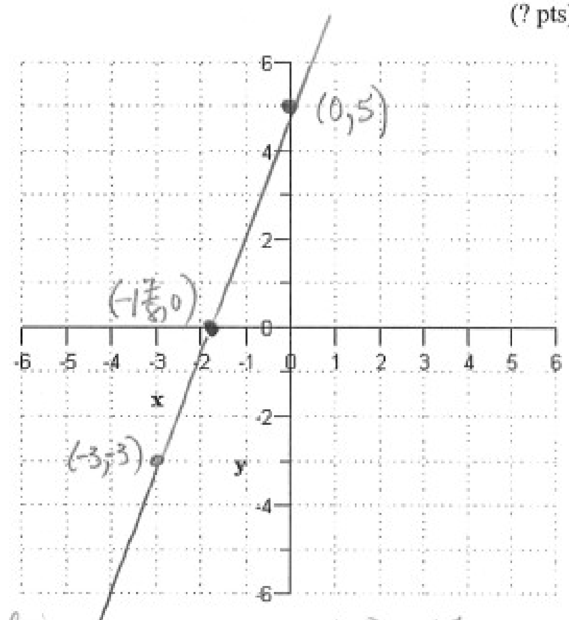
$\Rightarrow y = 5$. The y-intercept is $(0, 5.)$.

When $y = 0$, we have

$3(0) - 8x = 15$

$\Rightarrow -8x = 15 \Rightarrow x = \frac{15}{-8} = -1\frac{7}{8}$

The x-intercept is $(-\frac{15}{8}, 0)$.



Let $x = -3$; $3y - 8(-3) = 15$

$\Rightarrow 3y + 24 = 15 \Rightarrow 3y = -9 \Rightarrow y = -$

6. Solve the system

$$\begin{cases} 2x - y = 0 \\ 3x + 2y = 10 \\ x - y = -1 \end{cases}$$

by any method. (Show your work). (? pts)

First solve $\begin{cases} 2x - y = 0 \\ x - y = -1 \end{cases}$: $\begin{array}{l} 2x - y = 0 \\ -x + y = 1 \end{array}$ $\xrightarrow{\text{Adding}}$ $x = 1$.

So $y = 2x = 2(1) = 2$. Thus $(1, 2)$ solves the 1st + 3rd eqn.

Now check that it solves the 2nd equation:

$3x + 2y = 10$

$3(1) + 2(2) = 10$

$7 = 10$

Since $7 \ne 10$, $(1, 2)$ is not a soln of the 2nd eqn.

Hence, it is not a soln of the system.

7. What is meant by an inconsistent system? Describe the graph of an inconsistent system that consists of two equations in two variables.

A inconsistent system is a system of equations that does not have a solution. The graph of such a system consists of two parallel lines. (See Wilson p. 62)

8. Solve the following systems using the 3 operations of equivalent systems and labeling each operation as we did in class. If there are no solutions, then state: "There are no solutions." If there are infinitely many solutions, then express them in terms of a parameter t (such as $x = 2t - 1$, $y = t - 5$, $z = t$) and then give any two particular solutions. (? pts)

(a) $\begin{cases} 3x - 3y + z = 5 \\ x \quad + y + z = 11 \\ 2x + \qquad 2z = 10 \end{cases}$ $\xrightarrow{E_1 \leftrightarrow E_2}$ $\begin{matrix} x + y + z = 11 \\ 3x - 3y + z = 5 \\ 2x \qquad 2z = 10 \end{matrix}$ $\xrightarrow[-2E_1 + E_3 \to E_3]{-3E_1 + E_2 \to E_2}$ $\begin{matrix} x + y + z = 11 \\ -6y - 2z = -28 \\ -2y \qquad = -12 \end{matrix}$

$\xrightarrow{-\frac{1}{2}E_3}$ $\begin{matrix} x + y + z = 11 \\ -6y - 2z = -28 \\ y \qquad = 6 \end{matrix}$ $\xrightarrow{6E_3 + E_2 \to E_2}$ $\begin{matrix} x + y + z = 11 \\ -2z = 8 \\ y = 6 \end{matrix}$

$\xrightarrow{\frac{1}{2}E_2 \to E_2}$ $\begin{matrix} x + y + z = 11 \\ z = -4 \\ y = 6 \end{matrix}$ $\Rightarrow x = 11 - y - z = 11 - 6 - (-4) = 9$

Thus, $x = 9$, $y = 6$, $z = -4$.

(b) $\begin{cases} x - \qquad z = 1 \\ 2x + 3y + z = 8 \\ x + y \qquad = 3 \end{cases}$ $\xrightarrow[-1 \cdot E_1 + E_3 \to E_3]{-2E_1 + E_2 \to E_2}$ $\begin{matrix} x \quad - z = 1 \\ 3y + 3z = 6 \\ y + z = 2 \end{matrix}$

$\xrightarrow{\frac{1}{3}E_2 \to E_2}$ $\begin{matrix} x \quad - z = 1 \\ y + z = 2 \\ y + z = 2 \end{matrix}$

$\xrightarrow{-1 \cdot E_2 + E_3 \to E_3}$ $\begin{matrix} x \quad - z = 1 \\ y + z = 2 \\ 0 = 0 \end{matrix}$

Thus, $x = 1 + z$ and $y = 2 - z$.

Let $z = t$ where t is any real number.

Hence there are infinitely many solutions of the form $x = 1 + t$, $y = 2 - t$, $z = t$

Particular solns: $t = 0 \Rightarrow x = 1, y = 2, z = 0$ $\quad t = -1$: $x = 0, y = 3, z = -1$

Part 2. Use your calculator (you may not borrow one during the exam) ~~on this part~~ for this part of the ~~test~~ exam.

9. Determine whether the cost of filling a gas tank as given in the table below is directly proportional to the number of gallons put into the tank. If it is, find a linear function that models the data. (? pts)

Number of gallons (g)	Cost(C) in dollars
4.50	5.40
6.23	7.47
17.21	20.63

$\frac{5.40}{4.5} = 1.2$, $\frac{7.47}{6.23} = 1.19904$, $\frac{20.63}{17.21} = 1.19872$

Rounding to the nearest cent, it is directly proportional where the linear function is

$C(g) = 1.199g$.

10. In a study of how the amount of milk protein from a cow varies with milk production, the following data was reported for Holstein-Friesan cows. Find the equation of the line of best fit. (10 pts)

Milk Production (kg/day)	Milk Protein (kg/day)
42	1.20
40	1.16
38	1.07
37	1.13
32	1.07
28	1.07

Following the directions on pp. 33-34, we get

$y = 0.008429x + 0.811827$

where x is the milk production in kg/day and y is the milk protein in kg/day.