

## JENSEN'S INEQUALITY AND LIAPUNOV'S DIRECT METHOD

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A common hypothesis of researchers investigating the behavior of solutions starting near zero of a system of ordinary differential equations

$$(0.1) \quad x'(t) = f(t, x), \quad \text{where } f(t, 0) = 0,$$

is that  $f$  is bounded when  $x$  is bounded. With this hypothesis, Marachkov proved in 1940, using an annulus argument together with Liapunov's direct method, that the zero solution of (0.1) is asymptotically stable if a positive definite Liapunov function for (0.1) exists with negative definite derivative. However, the hypothesis excludes some properties that actually promote asymptotic stability. The point of this paper is to derive an alternative to the annulus argument for functional differential equations

$$(0.2) \quad x'(t) = F(t, x_t), \quad \text{where } F(t, 0) = 0,$$

which does not require Marachkov's hypothesis. It is based on finding conditions that not only result in a suitable wedge bounding the derivative of a Liapunov functional  $V$  for (0.2) but one that is also convex downward. The convexity makes it then possible to use Jensen's inequality to obtain an upper bound on  $V$  from which we can obtain stability results, such as finding conditions so that all solutions of the scalar equation

$$(0.3) \quad x' = -a(t)x + b(t)x(t-1) - x^3$$

approach zero, without requiring  $a$  or  $b$  be bounded.

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