STABILITY, FIXED POINTS, AND INVERSES OF DELAYS

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Abstract. The scalar equation (1) $x'(t) = -\int_{t-r(t)}^{t} a(t,s)g(x(s))\,ds$ with variable delay $r(t) \geq 0$ is investigated, where $t - r(t)$ is increasing and $xg(x) > 0$ ($x \neq 0$) in a neighborhood of $x = 0$. We find conditions for r, a, and g so that for a given continuous initial function ψ a mapping P for (1) can be defined on a complete metric space C_ψ and in which P has a unique fixed point. The end result is not only conditions for the existence and uniqueness of solutions of (1) but also for the stability of the zero solution. We also find conditions ensuring the zero solution is asymptotically stable by changing to an exponentially weighted metric on a closed subset of C_ψ. Finally, we parlay the methods for (1) into results for (2) $x'(t) = -\int_{t-r(t)}^{t} a(t,s)g(s,x(s))\,ds$ and (3) $x'(t) = -a(t)g(x(t-r(t)))$.

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