# Function Bounds for Solutions of Volterra Equations and Exponential Asymptotic Stability

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**Abstract**

An extension of Gronwall's inequality is derived for the purpose of finding functions that bound the solutions of the scalar linear Volterra integro-differential equation

$$x'(t) = -a(t)x(t) + \int_0^t b(t-s)x(s)\,ds,$$

where $a, b\colon [0,\infty) \to \mathbf{R}$ are continuous functions and $b$ is nonnegative. These functions are used to search for conditions that drive solutions to zero and conditions under which the zero solution is exponentially asymptotically stable.

*Key words:* Exponential asymptotic stability, function bounds, Gronwall's inequality, variation of parameters formula, Volterra integro-differential equations

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