Maple's Numeric Integration

```
> restart;
```

We are going to look at Maple's built-in numeric integration. We begin by going to the help page for numerical integration .

```
> ?evalf,int;
```

After scanning the page, we set infolevel to 5 for both evalf/int and int and look at an example on the help page in some detail.

```
> infolevel[`evalf/int`]:=5;infolevel[int]:=5;
```

$$infolevel_{evalf/int} := 5$$

$$infolevel_{int} := 5$$

We look at a first integral.

```
> Int( exp(-x^3)/(x^2+1), x = 0..1 )=int( exp(-x^3)/(x^2+1), x = 0..1
  );
int: Beginning integration with _EnvContinuous=_EnvContinuous,
_EnvAllSolutions=_EnvAllSolutions, and _EnvCauchyPrincipalValue=
_EnvCauchyPrincipalValue.
Definite Integration:   Integrating expression on x=0..1
Definite Integration:   Using the integrators [distribution, piecewise,
series, o, polynomial, ln, lookup, cook, ratpoly, elliptic,
elliptictrig, meijergspecial, improper, asymptotic, ftoc, ftocms,
meijerg, contour]
Definite Integration:   Trying method distribution.
Definite Integration:   Trying method piecewise.
Definite Integration:   Trying method series.
Definite Integration:   Trying method o.
Definite Integration:   Trying method polynomial.
Definite Integration:   Trying method ln.
Definite Integration:   Trying method lookup.
LookUp Integrator:   unable to find the specified integral in the table
Definite Integration:   Trying method cook.
Cook LookUp Integrator:   Given Integral
Int(exp(-x^3)/(x^2+1),x = 0 .. 1)
Fits into this pattern:
Int(exp(-Ucplex*x^S1)*x^N*ln(B*x^DL)^M*cos(C1*x^R)/((A0+A1*x^D)^P),x =
t1 .. t2)

Cook LookUp Integrator:   --> but does not fit into any sub-classes
Cook LookUp Integrator:   returning answer from cook pattern 1c
Definite Integration:   Trying method ratpoly.
Definite Integration:   Trying method elliptic.
int/elliptic: trying elliptic integration
int/ellalg/elltype: Checking for an elliptic integral exp(-x^3)/(x^2+1)
freeof(x) x
Definite Integration:   Trying method elliptictrig.
Definite Integration:   Trying method meijergspecial.
Definite Integration:   Trying method improper.
Definite Integration:   Trying method asymptotic.
Definite Integration:   Trying method ftoc.
int/indef1: first-stage indefinite integration
int/indef2: second-stage indefinite integration
int/exp: case of integrand containing exp
int/indef1: first-stage indefinite integration
```

int/indef2: second-stage indefinite integration
int/exp: case of integrand containing exp
int/prpexp: case ratpoly*exp(arg)
int/rpexp: nonlinear denominator case not yet coded
int/prpexp: case ratpoly*exp(arg)
int/rischnorm: enter Risch-Norman integrator
int/rischnorm: exit Risch-Norman integrator
int/risch: enter Risch integration
int/risch: the field extensions are

$$\left[x\sim,\ e^{-x\sim^3}\right]$$

int/risch: Introduce the namings:

$$\left\{\_th_1 = e^{-x\sim^3}\right\}$$

int/risch/int: integrand is

$$\frac{\_th_1}{x\sim^2+1}$$

int/risch/exppoly: integrating

$$\frac{\_th_1}{x\sim^2+1}$$

int/risch/diffeq: solving Risch d.e. y' + f y = g where f,g are:

$$-3\,x\sim^2,\ \frac{1}{x\sim^2+1}$$

int/risch/DEratpoly: solving Risch d.e. y' + f y = g where f,g are:

$$-3\,x\sim^2,\ \frac{1}{x\sim^2+1}$$

int/risch/exppoly: Risch d.e. has no solution
int/risch: exit Risch integration
Definite Integration: Trying method ftocms.
int/risch: enter Risch integration
int/risch: the field extensions are

$$\left[x\sim,\ e^{-x\sim^3}\right]$$

int/risch: Introduce the namings:

$$\left\{\_th_1 = e^{-x\sim^3}\right\}$$

int/risch/int: integrand is

$$\frac{\_th_1}{x\sim^2+1}$$

int/risch/exppoly: integrating

$$\frac{\_th_1}{x\sim^2+1}$$

int/risch/diffeq: solving Risch d.e. y' + f y = g where f,g are:

$$-3\,x\sim^2,\ \frac{1}{x\sim^2+1}$$

int/risch/DEratpoly: solving Risch d.e. y' + f y = g where f,g are:

$$-3\,x\sim^2,\ \frac{1}{x\sim^2+1}$$

```
int/risch/exppoly: Risch d.e. has no solution
int/risch: exit Risch integration
Definite Integration:   Trying method meijerg.
Definite Integration:   Trying method contour.
int/definite/contour: contour integration
Definite Integration:   Returning integral unevaluated.
```

$$\int_0^1 \frac{e^{-x^3}}{x^2+1}\,dx = \int_0^1 \frac{e^{-x^3}}{x^2+1}\,dx$$

We see that Maple tries many things but cannot do an exact evaluation, so we now try to integrate numerically usings Maple's default method.

```
> Int( exp(-x^3)/(x^2+1), x = 0..1 )=evalf(Int( exp(-x^3)/(x^2+1), x
  = 0..1 ));
evalf/int/control: integrating on 0 .. 1 the integrand
```

$$\frac{e^{-x^3}}{x^2+1}$$

```
evalf/int/control: tolerance = .5000000000e-9; method = _DEFAULT;
maxintervals = _DEFAULT
Control: Entering NAGInt
Control: trying d01ajc (nag_1d_quad_gen)
d01ajc: epsabs=.500000000000000e-12; epsrel=.5000000000e-9;
max_num_subint=200
d01ajc: procedure for evaluation is:
proc (x) exp(-1.*x^3)/(x^2+1.) end proc
d01ajc: trying evalhf callbacks
d01ajc: result=.664936943073927478
d01ajc: abserr=.369114152062270785e-14; num_subint=1; fun_count=21
Control: result=.664936943073927478
```

$$\int_0^1 \frac{e^{-x^3}}{x^2+1}\,dx = 0.6649369431$$

We have success. Maple used **d01agc**, a **NAG (Numerical Algorithms Group)** routine that operates at hardware floating-point speed. Next we use an adaptive double exponential method. We also ask for 20 digits of precision.

```
> Int( exp(-x^3)/(x^2+1), x = 0..1 )=evalf(Int( exp(-x^3)/(x^2+1), x
  = 0..1,digits=20, method=_Dexp));
evalf/int/control: integrating on 0 .. 1 the integrand
```

$$\frac{e^{-x^3}}{x^2+1}$$

```
evalf/int/control: tolerance = .5000000000e-19; method = _Dexp;
maxintervals = _DEFAULT
evalf/int/CreateProc: Trying makeproc
evalf/int/control: procedure for evaluation is:
proc (x) local xl; try evalf(exp(-x^3)/(x^2+1)) catch "function does
not evaluate to numeric": error catch: evalf(limit(exp(-xl^3)/(xl^2+1),
xl = x,'real')) end try end proc
evalf/int/quadexp: applying double-exponential method
evalf/int/samp_quad: Delta[1] = .945928887853066226922362
evalf/int/samp_quad: Delta[2] = -.27718010884060901134870 9
evalf/int/samp_quad: result = .676717885526439657361269, HError =
.27718010884060901134870 9
evalf/int/samp_quad: Delta[3] = -.11760689773320163014201e-1
```

```
evalf/int/samp_quad: result = .66495719575311949434706 8, HError =
.11760689773320163014201e-1
evalf/int/samp_quad: Delta[4] = -.20252696217136101614e-4
evalf/int/samp_quad: result = .66493694305690235824545 4, HError =
.20252696217136101614e-4
evalf/int/samp_quad: Delta[5] = .17025097882968e-10
evalf/int/samp_quad: result = .66493694307392745612842 2, HError =
.17025097882968e-10
evalf/int/samp_quad: Delta[6] = .2477e-20
evalf/int/samp_quad: result = .66493694307392745613089 9, HError =
.2477e-20
evalf/int/samp_quad: result = .66493694307392745613089 9, HError =
.2477e-20
evalf/int/quadexp: errest = .16055910452277686044741e-19, AbsError =
.50000000000000000000000e-22, RelError = .5000000000e-19
From quadexp, result = .66493694307392745613089 9 integrand evals = 136
error = .16055910452277686044741e-19
tolerance = .33246847153696372806545e-19
```

$$\int_0^1 \frac{e^{-x^3}}{x^2+1}\,dx = 0.66493694307392745613$$

Now we will try an adaptive Gaussian quadrature method and, after raising Digits to 25, set $10^{(-20)}$ as a relative error tolerance.

```
> Digits:=25;
```

$$Digits := 25$$

```
> Int( exp(-x^3)/(x^2+1), x = 0..1 )=evalf(Int( exp(-x^3)/(x^2+1), x
  = 0..1,epsilon=1.0e-20, method=_Gquad));
evalf/int/control: integrating on 0 .. 1 the integrand
```

$$\frac{e^{-x^3}}{x^2+1}$$

```
evalf/int/control: tolerance = .10e-19; method = _Gquad; maxintervals =
_DEFAULT
evalf/int/CreateProc: Trying makeproc
evalf/int/control: procedure for evaluation is:
proc (x) local xl; try evalf(exp(-x^3)/(x^2+1)) catch "function does
not evaluate to numeric": error catch: evalf(limit(exp(-xl^3)/(xl^2+1),
xl = x,'real')) end try end proc
evalf/int/AGQ: applying adaptive Gaussian quadrature
From _Gquad, result = .66493694307392745613094 0 8 integrand evals =
98 error = .107e-25
tolerance = .66493694307392745613094 0 8e-20
```

$$\int_0^1 \frac{e^{-x^3}}{x^2+1}\,dx = 0.66493694307392745613094\,08$$