Adaptive Quadrature

```
> restart;
```

nalib

```
> libname:="c:/nalib",libname;
```

libname := "/nalib", "/Library/Frameworks/Maple.framework/Versions/15/lib", "/Library/Frameworks/Maple.framework/Versions/15/toolbox/NAG/lib"

```
> with(numanal);
```

[*SOR*, *SOR_dir*, *adaptq*, *adaptq_dir*, *bezier*, *bezier_dir*, *bisection*, *bisection_dir*, *chop*, *chop_dir*, *clamped_spline*, *clamped_spline_dir*, *divided_diff*, *divided_diff_dir*, *extrap*, *extrap_dir*, *falseposition*, *falseposition_dir*, *fixedpoint*, *fixedpoint_dir*, *gaussseidel*, *gaussseidel_dir*, *hermite*, *hermite_dd*, *hermite_dd_dir*, *hermite_dir*, *horner*, *horner_dir*, *jacobi*, *jacobi_dir*, *muller*, *muller_dir*, *natural_spline*, *natural_spline_dir*, *newton*, *newton_dir*, *romberg*, *romberg_dir*, *secant*, *secant_dir*, *steffensen*, *steffensen_dir*]

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We enter the formula for **S** for **adaptive Simpson** as a Maple function.

```
> S:=(f,a,b)->(b-a)/6*(f(a)+4*f((a+b)/2)+f(b));
```

$$S := (f, a, b) \rightarrow \frac{1}{6}\,(b-a)\left(f(a) + 4\,f\left(\frac{1}{2}\,a + \frac{1}{2}\,b\right) + f(b)\right)$$

We enter our function as a Maple function.

```
> f:=x->x^2*ln(x);
```

$$f := x \rightarrow x^2 \ln(x)$$

We compute S(1,1.5) for f.

```
> S1:=S(f,1,1.5);
```

$$S1 := 0.1922453073$$

We compute S(1,1.25) for f.

```
> S2:=S(f,1,1.25);
```

$$S2 := 0.03937243405$$

We compute S(1.25,1.5) for f.

```
> S3:=S(f,1.25,1.5);
```

$$S3 := 0.1528860264$$

We compute our integral estimate.

```
> estimate:=S2+S3;
```

$$estimate := 0.1922584604$$

We compute our preliminary error estimate, which is approximately15 times the error estimate.

```
> errorest:=abs(S1-(S2+S3));
```

$$errorest := 0.0000131532$$

We compute the approximate error estimate.

```
> approxerror:=errorest/15;
```

$$approxerror := 8.768800000\ 10^{-7}$$

We compute the actual integral value.

```
> Int(f(x),x=1..1.5)=int(f(x),x=1..1.5);
```

$$\int_1^{1.5} x^2 \ln(x)\, dx = 0.1922593577$$

We compute the actual error in the approximation.

```
> errorr:=int(f(x),x=1..1.5)-S2-S3;
```

$$errorr := 8.972\ 10^{-7}$$

We are going to use **adaptive quadrature** with **Simpson's rule** to approximate $\int_1^3 \frac{100 \sin\left(\frac{10}{x}\right)}{x^2}\, dx$ with tolerance 10^{-4}. We will use the procedure **adaptq** in the class library **nalib**. We first check the directions for **adaptq**.

```
> adaptq_dir();
adaptq returns an approximation to a definite integral.

The arguments for adapt are:
(1)the function being integrated
(2)the lower limit of integration
(3)the upper limit of integration
(4)tolerance
(5)maximum number of levels
(6)variable for returning the integral approximation

If assigning the result to a variable, have the
variable and the 6th argument the same.

If S is the variable for returning the integral's value
and has already been given a value,
the procedure should be preceded by the statement:
S:='S'
```

We enter the function.

```
> f:=(100/x^2)*sin(10/x);
```

$$f := \frac{100 \sin\left(\frac{10}{x}\right)}{x^2}$$

We evaluate the approximation with **adaptq**.

```
> S:=adaptq(f,1,3,10^(-4),100,S);

The integral of F from   1.00000000 to   3.00000000 is
 -1.42601481 to within 1.00000000e-04
The number of function evaluations is: 93
```

$$S := -1.426014813$$

We find the actual value of the integral.

```
> actual:=evalf(int(f,x=1..3));
```

$$actual := -1.426024757$$

We find the actual error.

```
> err:=abs(S-actual);
```

$$err := 0.000009944$$

NumericalAnalysis

We do the same problem using the command AdaptiveQuadrature in the NumericalAnalysis package.

```
> with(Student[NumericalAnalysis]);
```

[*AbsoluteError*, *AdamsBashforth*, *AdamsBashforthMoulton*, *AdamsMoulton*, *AdaptiveQuadrature*, *AddPoint*, *ApproximateExactUpperBound*, *ApproximateValue*, *BackSubstitution*, *BasisFunctions*, *Bisection*, *CubicSpline*, *DataPoints*, *Distance*, *DividedDifferenceTable*, *Draw*, *Euler*, *EulerTutor*, *ExactValue*, *FalsePosition*, *FixedPointIteration*, *ForwardSubstitution*, *Function*, *InitialValueProblem*, *InitialValueProblemTutor*, *Interpolant*, *InterpolantRemainderTerm*, *IsConvergent*, *IsMatrixShape*, *IterativeApproximate*, *IterativeFormula*, *IterativeFormulaTutor*, *LeadingPrincipalSubmatrix*, *LinearSolve*, *LinearSystem*, *MatrixConvergence*, *MatrixDecomposition*, *MatrixDecompositionTutor*, *ModifiedNewton*, *NevilleTable*, *Newton*, *NumberOfSignificantDigits*, *PolynomialInterpolation*, *Quadrature*, *RateOfConvergence*, *RelativeError*, *RemainderTerm*, *Roots*, *RungeKutta*, *Secant*, *SpectralRadius*, *Steffensen*, *Taylor*, *TaylorPolynomial*, *UpperBoundOfRemainderTerm*, *VectorLimit*]

We find the approximation using **output=value**.

```
> ans:=AdaptiveQuadrature(f,x=1..3,method=simpson,output=value);
```

$$ans := -1.426014810$$

Since **method=simpson** is the default, that option can be skipped. We do this with **output=information**.

```
> AdaptiveQuadrature(f,x=1..3,output=information);
INTEGRAL: Int(100/x^2*sin(10/x),x=1..3) = -1.42602476
APPROXIMATION METHOD: Adaptive Simpson's Rule
---------------------------------- INFORMATION TABLE ------------------
----------------
    Approximate Value         Absolute Error         Relative Error
        -1.42601481                9.946e-06         0.0006975 %
---------------------------------- ITERATION HISTORY ------------------
---------------------
   Interval             Status         Present Stack
      1..3               fail           EMPTY
      1..2               fail           [2, 3]
      1..3/2             fail           [[1], [3/2, 2]]
      1..5/4             fail           [[2], [5/4, 3/2]]
      1..9/8             fail           [[3], [9/8, 5/4]]
      1..17/16           fail           [[4], [17/16, 9/8]]
      1..33/32           PASS           [[5], [33/32, 17/16]]
  33/32..17/16           PASS           [[4], [17/16, 9/8]]
  17/16..9/8             fail           [[3], [9/8, 5/4]]
  17/16..35/32           PASS           [[4], [35/32, 9/8]]
  35/32..9/8             PASS           [[3], [9/8, 5/4]]
    9/8..5/4             fail           [[2], [5/4, 3/2]]
    9/8..19/16           fail           [[3], [19/16, 5/4]]
    9/8..37/32           PASS           [[4], [37/32, 19/16]]
  37/32..19/16           PASS           [[3], [19/16, 5/4]]
  19/16..5/4             PASS           [[2], [5/4, 3/2]]
    5/4..3/2             fail           [[1], [3/2, 2]]
    5/4..11/8            fail           [[2], [11/8, 3/2]]
    5/4..21/16           PASS           [[3], [21/16, 11/8]]
  21/16..11/8            PASS           [[2], [11/8, 3/2]]
   11/8..3/2             fail           [[1], [3/2, 2]]
   11/8..23/16           PASS           [[2], [23/16, 3/2]]
  23/16..3/2             PASS           [[1], [3/2, 2]]
    3/2..2               fail           [2, 3]
```

```
   3/2..7/4          fail         [[1], [7/4, 2]]
   3/2..13/8         fail         [[2], [13/8, 7/4]]
   3/2..25/16        PASS         [[3], [25/16, 13/8]]
 25/16..13/8         PASS         [[2], [13/8, 7/4]]
  13/8..7/4          fail         [[1], [7/4, 2]]
  13/8..27/16        PASS         [[2], [27/16, 7/4]]
 27/16..7/4          PASS         [[1], [7/4, 2]]
   7/4..2            fail         [2, 3]
   7/4..15/8         PASS         [[1], [15/8, 2]]
  15/8..2            PASS         [2, 3]
     2..3            fail         EMPTY
     2..5/2          fail         [5/2, 3]
     2..9/4          fail         [[1], [9/4, 5/2]]
     2..17/8         PASS         [[2], [17/8, 9/4]]
  17/8..9/4          PASS         [[1], [9/4, 5/2]]
   9/4..5/2          fail         [5/2, 3]
   9/4..19/8         PASS         [[1], [19/8, 5/2]]
  19/8..5/2          PASS         [5/2, 3]
   5/2..3            fail         EMPTY
   5/2..11/4         PASS         [11/4, 3]
  11/4..3            PASS         EMPTY
-----------------------------------------------------------------------
---------------------
Number of Function Evaluations:     93
```

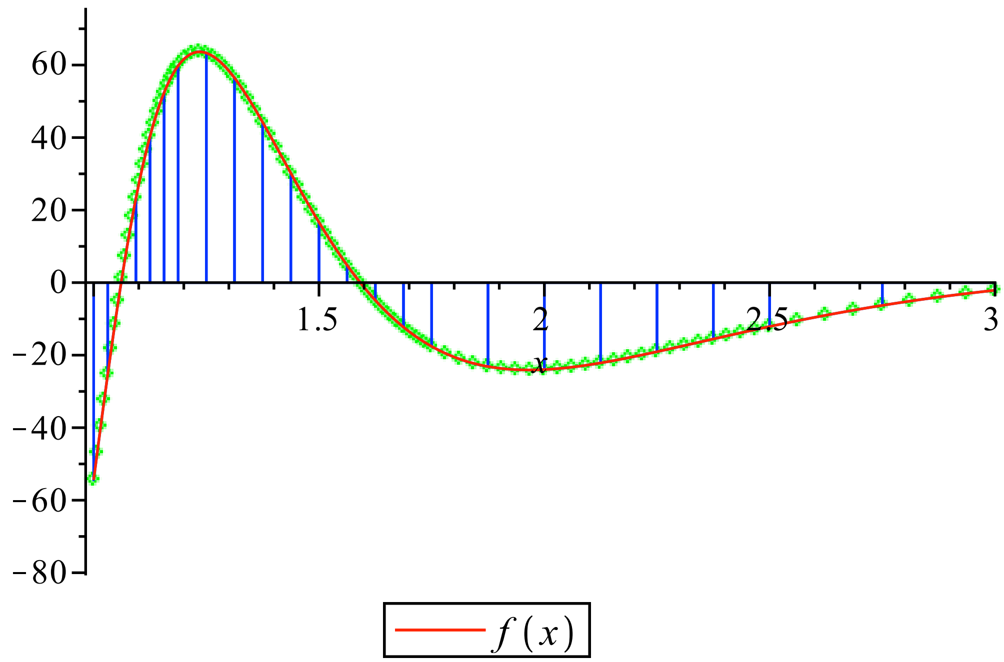
Compare the information above to the use of **output=plot.**

```
> AdaptiveQuadrature(f,x=1..3,output=plot);
```

An Approximation of the Integral of

$$f(x) = \frac{100 \sin\left(\frac{10}{x}\right)}{x^2}$$

on the Interval [1, 3]
Using Adaptive Simpson's rule
Integral Value: −1.426024756
Approximation: −1.426014810



Partitions: 23

The default tolerance here is 10^{-4}. To get a different tolerance, say 10^{-6}, we use **method=[simpson,10^(-6)]** instead of **method = simpson**.

```
> betterans:=AdaptiveQuadrature(f,x=1..3,method=[simpson,10^(-6)],
  output=value);
```

$$betterans := -1.426024678$$