Bisection Method

We begin by using the class maple library.

```
> restart;
> libname:="c:/nalib",libname;
```

libname := "/nalib", "/Library/Frameworks/Maple.framework/Versions/15/lib",
"/Library/Frameworks/Maple.framework/Versions/15/toolbox/NAG/lib"

```
> with(numanal);
```

[*SOR*, *SOR_dir*, *adaptq*, *adaptq_dir*, *bezier*, *bezier_dir*, *bisection*, *bisection_dir*, *chop*, *chop_dir*, *clamped_spline*, *clamped_spline_dir*, *divided_diff*, *divided_diff_dir*, *extrap*, *extrap_dir*, *falseposition*, *falseposition_dir*, *fixedpoint*, *fixedpoint_dir*, *gaussseidel*, *gaussseidel_dir*, *hermite*, *hermite_dd*, *hermite_dd_dir*, *hermite_dir*, *horner*, *horner_dir*, *jacobi*, *jacobi_dir*, *muller*, *muller_dir*, *natural_spline*, *natural_spline_dir*, *newton*, *newton_dir*, *romberg*, *romberg_dir*, *secant*, *secant_dir*, *steffensen*, *steffensen_dir*]

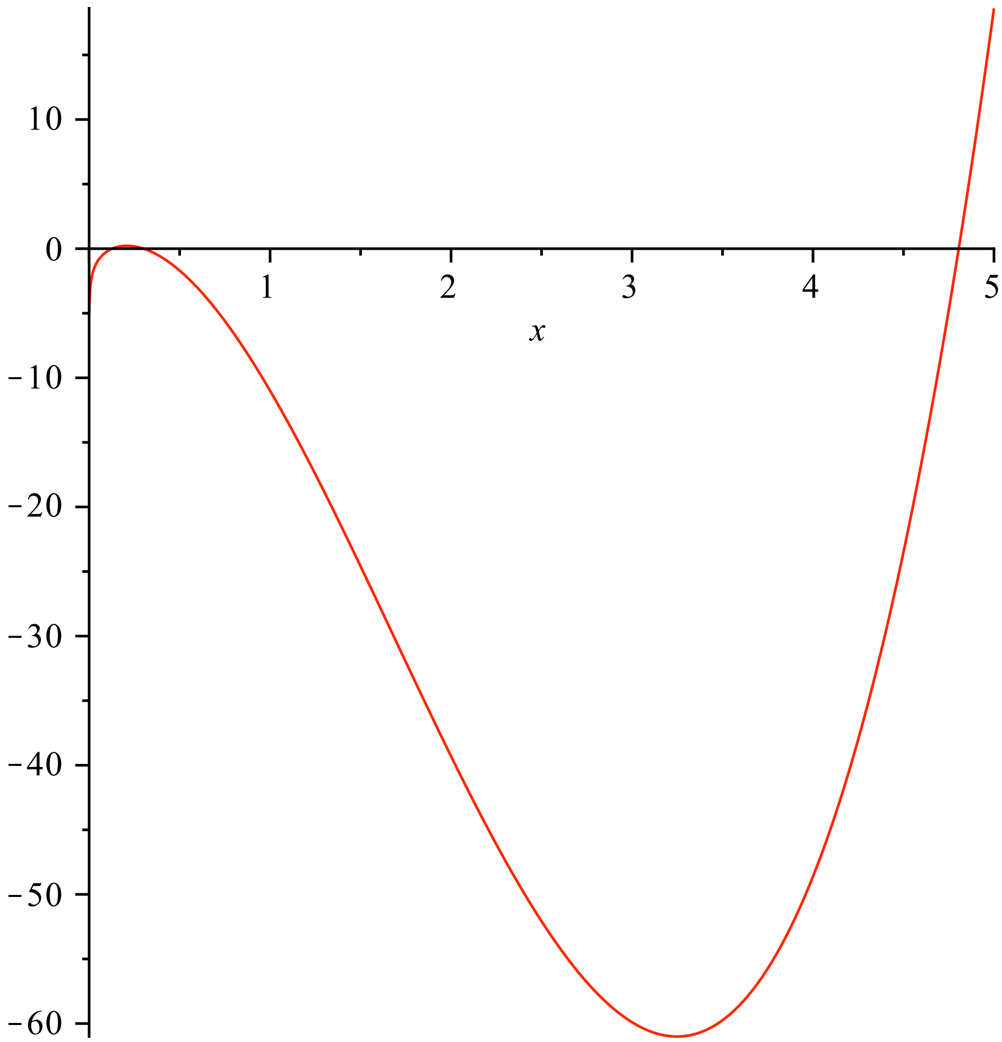
Suppose we wish to find all the roots of the following function accurate to within 10^{-6} .

```
> f:=4*x^3-20*x^2+3*x+2+ln(x);
```

$$f := 4\,x^3 - 20\,x^2 + 3\,x + 2 + \ln(x)$$

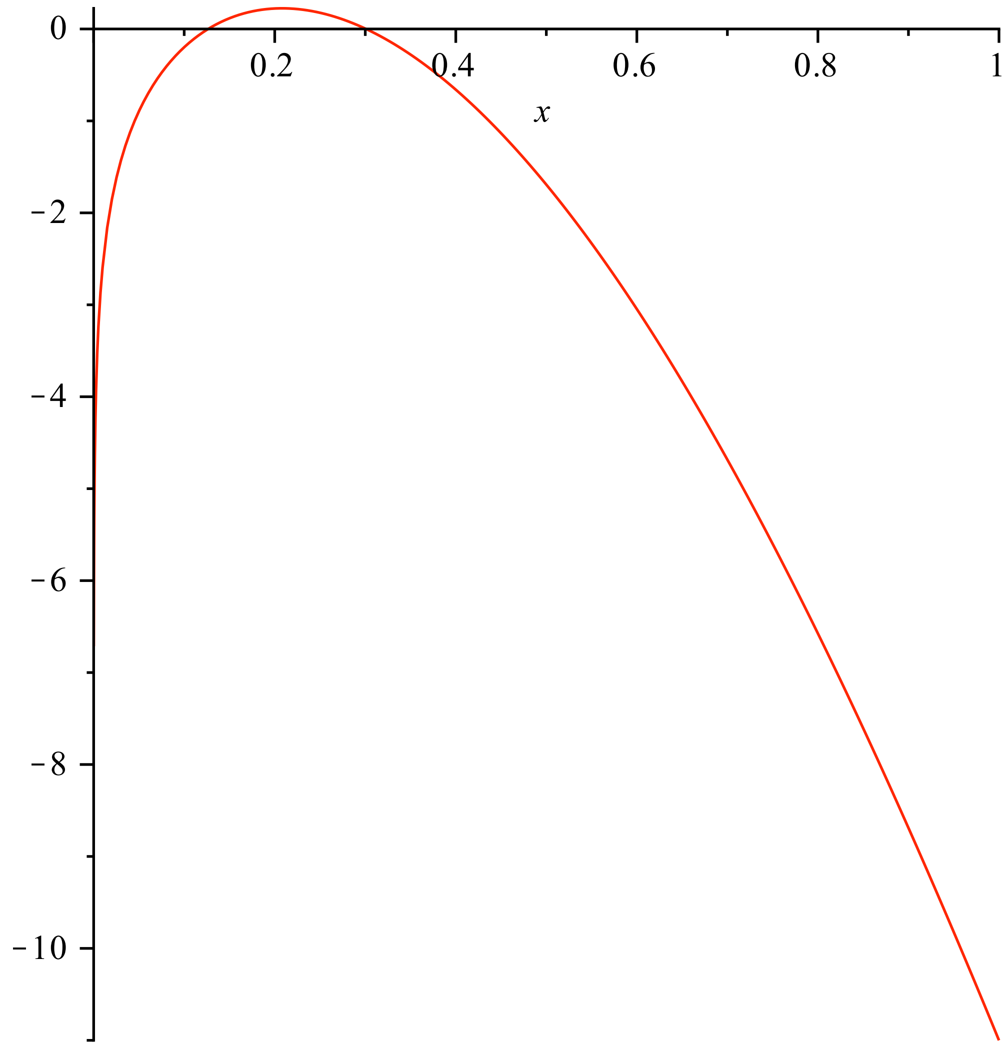
We plot the graph.

```
> plot(f,x=0..5);
```



We see we have a root near 4.8. On the other hand, it is difficult to see what is happening near 0, so we change our plot window.

```
> plot(f,x=0..1);
```



nalib

Now we see we also have roots near .12 and .3. We use the procedure **bisection** to find these roots. First, let's look at the directions for **bisection**.

```
> bisection_dir();
bisection returns a root of the given function.

The arguments for bisection are:
(1)function expression in x
(2)left end point
(3)right end point
(4)tolerance
(5)maximum number of iterations
(6)variable for returning root.

If assigning the result to a variable, have the
variable and the 6th argument the same.

If r is the variable for returning the root
and has already been given a value,
the procedure should be preceded by the statement:
r:='r'
```

We first find the root near .12. It is the only root between .1 and .14.

```
> r1:=bisection(f,.1,.14,.000001,100,r1);
```

```
  i     p                  f(p)
  -     -                  ----
  1     1.20000000e-01    -4.1351536e-02
  2     1.30000000e-01     2.0567171e-02
  3     1.25000000e-01    -9.1290420e-03
  4     1.27500000e-01     6.0267740e-03
  5     1.26250000e-01    -1.4732340e-03
  6     1.26875000e-01     2.2961210e-03
  7     1.26562500e-01     4.1629600e-04
  8     1.26406250e-01    -5.2725400e-04
  9     1.26484375e-01    -5.5175000e-05
 10     1.26523438e-01     1.8063700e-04
 11     1.26503906e-01     6.2750000e-05
 12     1.26494141e-01     3.7920000e-06
 13     1.26489258e-01    -2.5691000e-05
 14     1.26491699e-01    -1.0949000e-05
 15     1.26492920e-01    -3.5790000e-06
 16     1.26493530e-01     1.0600000e-07

The approximate solution is r1 =  0.12649353
with f(r1) =   0.00000011
```

$r1 := 0.1264935302$

The second root is the only one between .2 and .4.

```
> r2:=bisection(f,.2,.4,.000001,100,r2);
  i     p                  f(p)
  -     -                  ----
  1     3.00000000e-01     4.0271960e-03
  2     3.50000000e-01    -2.7832212e-01
  3     3.25000000e-01    -1.2411760e-01
  4     3.12500000e-01    -5.6705498e-02
  5     3.06250000e-01    -2.5493165e-02
  6     3.03125000e-01    -1.0520051e-02
  7     3.01562500e-01    -3.1930110e-03
  8     3.00781250e-01     4.3046900e-04
  9     3.01171875e-01    -1.3779300e-03
 10     3.00976562e-01    -4.7289500e-04
 11     3.00878906e-01    -2.1003000e-05
 12     3.00830078e-01     2.0478500e-04
 13     3.00854492e-01     9.1904000e-05
 14     3.00866699e-01     3.5454000e-05
 15     3.00872803e-01     7.2260000e-06
 16     3.00875854e-01    -6.8880000e-06
 17     3.00874329e-01     1.7000000e-07
 18     3.00875092e-01    -3.3610000e-06

The approximate solution is r2 =  0.30087509
with f(r2) =  -0.00000336
```

$r2 := 0.3008750915$

The final root is the only one between 4 and 5.

```
> r3:=bisection(f,4,5,.000001,100,r3);
  i     p                  f(p)
  -     -                  ----
  1     4.50000000e+00    -2.3495923e+01
  2     4.75000000e+00    -4.7543554e+00
  3     4.87500000e+00     6.3263077e+00
  4     4.81250000e+00     6.3859970e-01
  5     4.78125000e+00    -2.0943556e+00
  6     4.79687500e+00    -7.3704334e-01
  7     4.80468750e+00    -5.1518896e-02
```

```
  8    4.80859375e+00     2.9296533e-01
  9    4.80664062e+00     1.2057950e-01
 10    4.80566406e+00     3.4494225e-02
 11    4.80517578e+00    -8.5212350e-03
 12    4.80541992e+00     1.2984401e-02
 13    4.80529785e+00     2.2307880e-03
 14    4.80523682e+00    -3.1452230e-03
 15    4.80526733e+00    -4.5712200e-04
 16    4.80528259e+00     8.8683300e-04
 17    4.80527496e+00     2.1475500e-04
 18    4.80527115e+00    -1.2107900e-04
 19    4.80527306e+00     4.7038000e-05
 20    4.80527210e+00    -3.7020000e-05

The approximate solution is r3 =  4.80527210
with f(r3) =  -0.00003702
```

$$r3 := 4.805272102$$

Let's see what happens if we enter our bracketing numbers in the opposite order.

```
> r3:=bisection(f,5,4,.000001,100,r3);
Error, invalid input: bisection expects its 6th argument, root, to be of type name, but received 4.805272102
```

Looks like we forgot our directions.

```
> r3:='r3';
```

$$r3 := r3$$

```
> r3:=bisection(f,5,4,.000001,100,r3);
  i    p                  f(p)
  -    -                  ----
  1    4.50000000e+00    -2.3495923e+01
  2    4.75000000e+00    -4.7543554e+00
  3    4.87500000e+00     6.3263077e+00
  4    4.81250000e+00     6.3859970e-01
  5    4.78125000e+00    -2.0943556e+00
  6    4.79687500e+00    -7.3704334e-01
  7    4.80468750e+00    -5.1518896e-02
  8    4.80859375e+00     2.9296533e-01
  9    4.80664062e+00     1.2057950e-01
 10    4.80566406e+00     3.4494225e-02
 11    4.80517578e+00    -8.5212350e-03
 12    4.80541992e+00     1.2984401e-02
 13    4.80529785e+00     2.2307880e-03
 14    4.80523682e+00    -3.1452230e-03
 15    4.80526733e+00    -4.5712200e-04
 16    4.80528259e+00     8.8683300e-04
 17    4.80527496e+00     2.1475500e-04
 18    4.80527115e+00    -1.2107900e-04
 19    4.80527306e+00     4.7038000e-05
 20    4.80527210e+00    -3.7020000e-05

The approximate solution is r3 =  4.80527210
with f(r3) =  -0.00003702
```

$$r3 := 4.805272102$$

Looks like the procedure can take the endpoints in any order. Let's see what happens if we look for a root in the interval [.2, 5].

```
> r:=bisection(f,.2,5,.000001,100,r);
Error, (in bisection) The function values at a and b must have opposite signs
```

The procedure watches out for us. It is important to choose starting intervals that contain a single root.

NumericalAnalysis

There is also a Bisection command in the Student[NumericalAnalysis] package.

```
> with(Student):with(NumericalAnalysis):
```

We use the Bisection command on the above problem, looking for the root between 4 and 5. For just the root, we use

```
> Bisection(f,x=[4,5],stoppingcriterion=absolute,tolerance=10^(-6));
```

For the sequence of intervals along with the solution:

```
> Bisection(f,x=[4,5],stoppingcriterion=absolute,tolerance=10^(-6),
  output=sequence,maxiterations=100);
```

For even more information:

```
> Bisection(f,x=[4,5],stoppingcriterion=absolute,tolerance=10^(-6),
  output=information,maxiterations=100);
```

For a plot of what is going on:

```
> Bisection(f,x=[4,5],stoppingcriterion=absolute,tolerance=10^(-6),
  output=plot,maxiterations=100);
```

For a plot that can be animated:

```
> Bisection(f,x=[4,5],stoppingcriterion=absolute,tolerance=10^(-6),
  output=animation,maxiterations=100);
```