Muller's Method

nalib

```
> restart;
> libname:="c:/nalib",libname;
```

libname := "/nalib", "/Library/Frameworks/Maple.framework/Versions/15/lib", "/Library/Frameworks/Maple.framework/Versions/15/toolbox/NAG/lib"

```
> with(numanal);
```

[*SOR*, *SOR_dir*, *adaptq*, *adaptq_dir*, *bezier*, *bezier_dir*, *bisection*, *bisection_dir*, *chop*, *chop_dir*, *clamped_spline*, *clamped_spline_dir*, *divided_diff*, *divided_diff_dir*, *extrap*, *extrap_dir*, *falseposition*, *falseposition_dir*, *fixedpoint*, *fixedpoint_dir*, *gaussseidel*, *gaussseidel_dir*, *hermite*, *hermite_dd*, *hermite_dd_dir*, *hermite_dir*, *horner*, *horner_dir*, *jacobi*, *jacobi_dir*, *muller*, *muller_dir*, *natural_spline*, *natural_spline_dir*, *newton*, *newton_dir*, *romberg*, *romberg_dir*, *secant*, *secant_dir*, *steffensen*, *steffensen_dir*]

We apply Muller's method to the polynomial

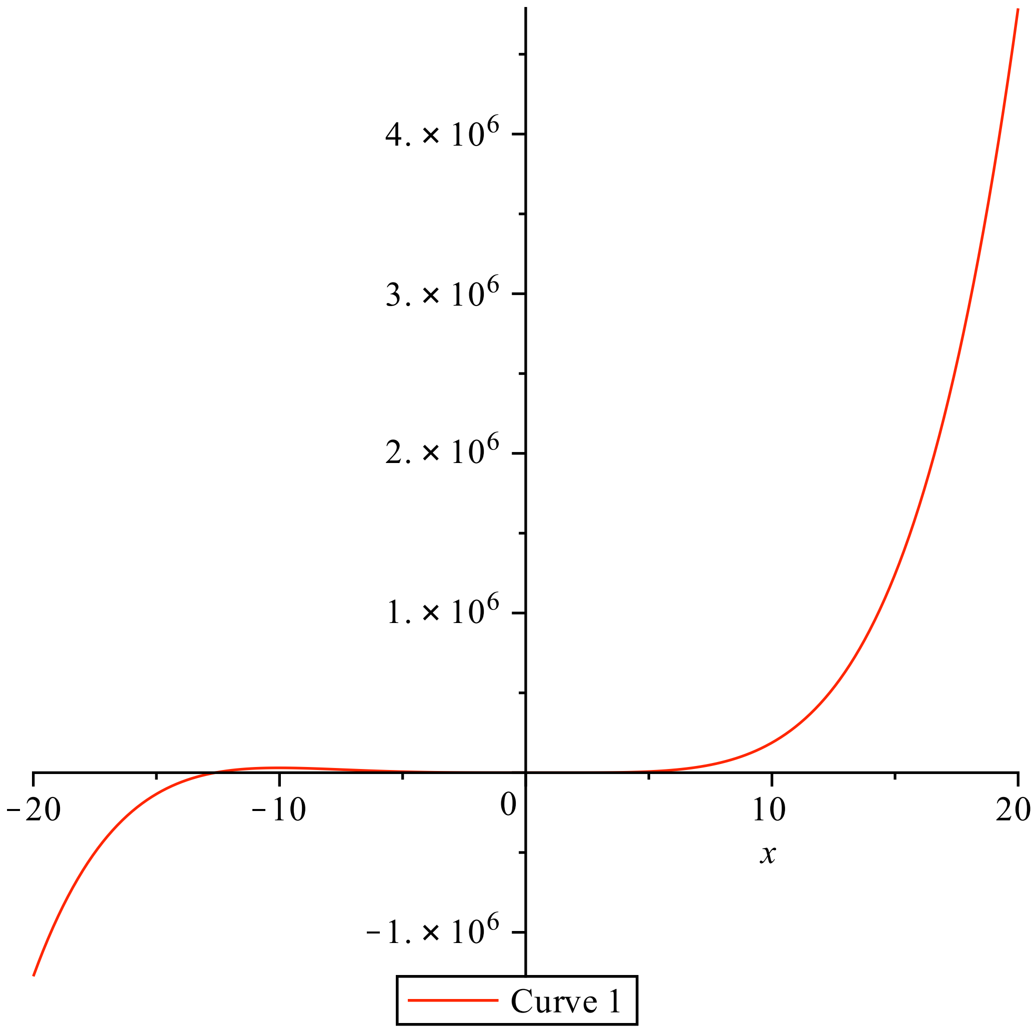
$$p(x) = x^5 + 11x^4 - 21x^3 - 10x^2 - 21x - 5\,.$$

We enter it as a function and plot its graph.

```
> p:=x^5+11*x^4-21*x^3-10*x^2-21*x-5;
```

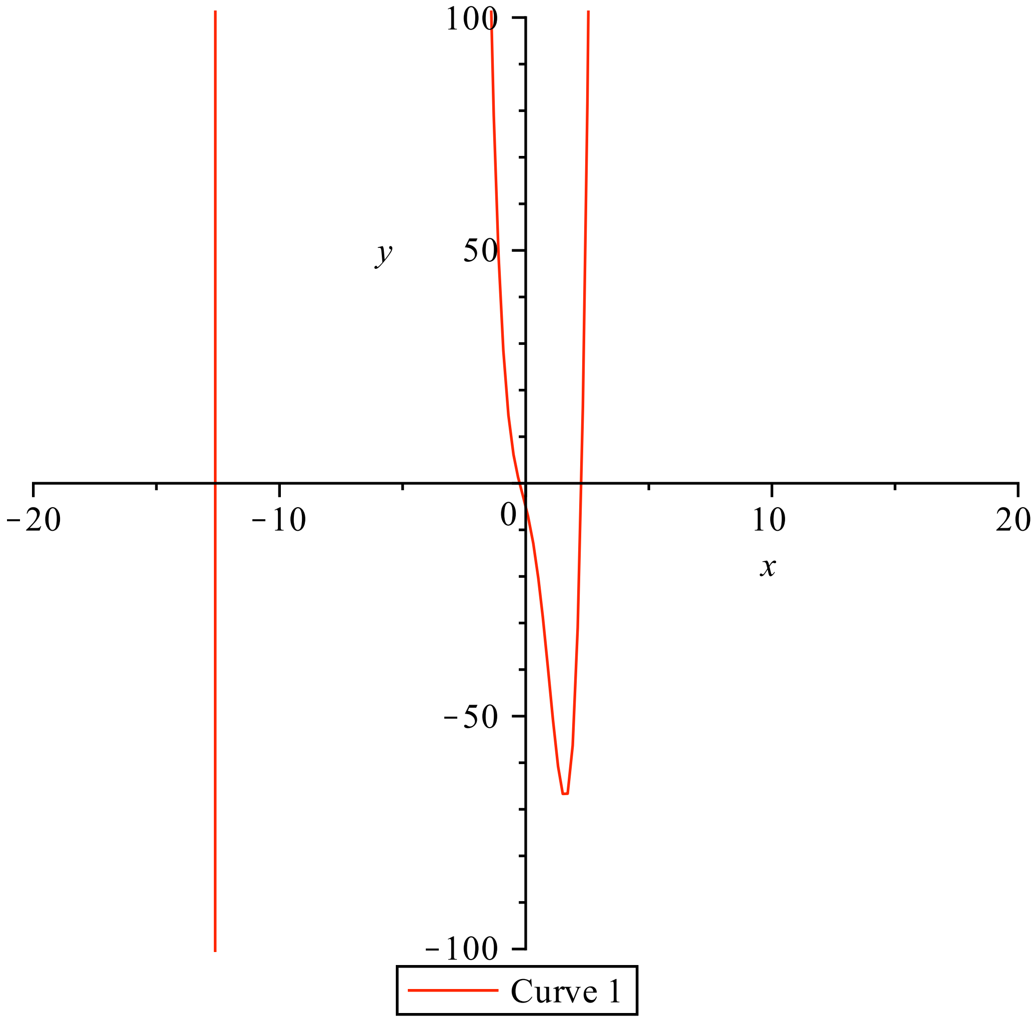
$$p := x^5 + 11x^4 - 21x^3 - 10x^2 - 21x - 5$$

```
> plot(p,x=-20..20);
```



We see the genearal picture, but we need to close down on the y -axis.

```
> plot(p,x=-20..20,y=-100..100);
```



OK, it looks like there are real roots near -12, 0, and 2. For **muller**, we need three initial approximations. For the first root, let's use -13, -12, and -11. Let's first look at the directions for **muller**.

```
> muller_dir();
muller returns a root of the given function.

The arguments for muller are:
(1)function expression in x (may include complex numbers)
(2)first initial approximation (real or complex)
(3)second initial approximation (real or complex)
(4)third initial approximation (real or complex)
(5)tolerance
(6)maximum number of iterations
(7)variable for returning root

If assigning the result to a variable, have the
variable and the 7th argument the same.

If r is the variable for returning the root
and has already been given a value,
the procedure should be preceded by the statement:
r:='r'
> r1:='r1';
```

$$r1 := r1$$

```
> r1:=muller(p,-13,-12,-11,.000001,100,r1);
  i    p                                    f(p)
  -    -                                    ----
  0   -13.                                 -12407.
  1   -12.                                 14359.
  2   -11.                                 26967.
  3   -12.59994181                         353.9589220
  4   -12.61283355                         -11.4975874
  5   -12.61242926                         .75265e-2
  6   -12.61242952                         .859e-4

The approximate solution is r1 = -12.61242952
with f(r1) = .859e-4
```

$$r1 := -12.61242952$$

Success! Now lets continue to the other real roots.

```
> r2:='r2';
```

$$r2 := r2$$

```
> r2:=muller(p,-1,0,1,.000001,100,r2);
  i    p                                    f(p)
  -    -                                    ----
  0   -1.                                  37.
  1   0.                                   -5.
  2   1.                                   -45.
  3   -.121590627                          -2.554311662
  4   -.2927492137                         .896229304
  5   -.2442568929                         -.122907603
  6   -.2501889361                         -.989348e-3
  7   -.2502369215                         -.388e-6
  8   -.2502369403                         -.1e-8

The approximate solution is r2 = -.2502369403
with f(r2) = -.1e-8
```

$$r2 := -0.2502369403$$

Again success.

```
> r3:='r3';
```

$$r3 := r3$$

```
> r3:=muller(p,1,2,3,.000001,100,r3);
  i    p                                    f(p)
  -    -                                    ----
  0   1.                                   -45.
  1   2.                                   -47.
  2   3.                                   409.
  3   2.175852055                          -19.03930431
  4   2.252059261                          -1.99058763
  5   2.259807679                          -.6955302e-1
  6   2.260085624                          .2412e-4
  7   2.260085528                          -.13e-6

The approximate solution is r3 = 2.260085528
with f(r3) = -.13e-6
```

$$r3 := 2.260085528$$

A third success. Now let's go for the complex roots. It's quite a bit harder to pick three sensible values because we don't have the graph to help us. But we will pick three complex numbers.

```
> r4:='r4';
```

$$r4 := r4$$

```
> r4:=muller(p,1+I,2+2*I,3+3*I,.000001,100,r4);
  i    p                                 f(p)
  -    -                                 ----
  0    1.+1.*I                           -32.-87.*I
  1    2.+2.*I                           -543.-586.*I
  2    3.+3.*I                           -3470.-2349.*I
  3    1.732495836+1.188553656*I         -211.6487196-106.1624567*I
  4    2.020194564+.8777883058*I         -197.2847438+38.94853631*I
  5    2.051451524+.5188757122*I         -99.30811301+57.78148733*I
  6    2.103013307+.1165317714*I         -35.70277871+19.08563255*I
  7    2.219660687-.10368232e-2*I        -9.64103953-.2351195298*I
  8    2.259706879-.2101914947e-2*I      -.9610007e-1-.5258619821*I
  9    2.260091443+.6063673e-5*I         .148119e-2+.1518426973e-2*I
 10    2.260085528-.299122e-9*I          -.13e-6-.7490319357e-7*I
 11    2.260085529+.141102e-13*I         .31e-6+.3533337711e-11*I

The approximate solution is r4 = 2.260085529+.141102e-13*I
with f(r4) = .31e-6+.3533337711e-11*I
```

$$r4 := 2.260085529 + 1.41102\ 10^{-14}\ I$$

Treating $1.41102\ 10^{-14}$ as 0, this is just the third root, so we try again.

```
> r4:='r4';
```

$$r4 := r4$$

```
> r4:=muller(p,3+5*I,8-4*I,7+5*I,.000001,100,r4);
  i    p                                 f(p)
  -    -                                 ----
  0    3.+5.*I                           34.-17275.*I
  1    8.-4.*I                           -61965.-94060.*I
  2    7.+5.*I                           -91202.+25245.*I
  3    2.975794100+4.448442705*I         -2351.947171-11668.57006*I
  4    2.887182309+4.048419154*I         -2673.263068-8230.489935*I
  5    1.703054080+3.891299736*I         2537.454577-3910.632812*I
  6    .7433560236+3.365954792*I         2042.032346-397.0723742*I
  7    .1878108880+2.874485318*I         966.2852151+413.6741360*I
  8    -.2984519774+2.360945117*I        216.0741610+459.4063189*I
  9    -.6069061498+1.845826256*I        -68.77850702+222.1724074*I
 10    -.7559605212+1.381307067*I        -91.64881755+51.12232600*I
 11    -.7847551064+.9711722882*I        -45.33653323-15.55441302*I
 12    -.7189332670+.6218485414*I        -8.81572832-22.19644598*I
 13    -.6061782181+.3337061282*I        4.643220372-12.09334258*I
 14    -.4830118358+.1174215908*I        5.272283224-3.542982071*I
 15    -.3593749336-.125581718e-1*I      2.404270722+.3001674485*I
 16    -.2511960793-.2808051508e-1*I     .12030757e-1+.5785730444*I
 17    -.2496715106+.130932762e-2*I      -.11667052e-1-.2697089218e-1*I
 18    -.2502303461-.2119744e-5*I        -.135907e-3+.4368782018e-4*I
 19    -.2502369406+.286412e-9*I         .6e-8-.5902974109e-8*I
 20    -.2502369403-.5288e-15*I          -.1e-8+.1089861007e-13*I

The approximate solution is r4 = -.2502369403-.5288e-15*I
with f(r4) = -.1e-8+.1089861007e-13*I
```

$$r4 := -0.2502369403 - 5.288\ 10^{-16}\ I$$

Treating $-5.288\ 10^{-16}$ as 0, this is just the third root, so we try again.

```
> r4:='r4';
```

$$r4 := r4$$

```
> r4:=muller(p,15+5*I,8-14*I,7+15*I,.000001,100,r4);
  i    p                                 f(p)
  -    -                                 ----
```

```
  0   15.+5.*I                          105430.+1577645.*I
  1   8.-14.*I                          294795.+1583470.*I
  2   7.+15.*I                          966198.-1505265.*I
  3   11.53092514+11.24272244*I         -1492624.426-745786.9056*I
  4   7.214159620+10.77770009*I         -105270.3476-590340.0399*I
  5   3.609875238+11.08048961*I         304978.2934-175551.9115*I
  6   2.226619469+9.725980630*I         170862.2865-27155.07649*I
  7   .408599268+8.558246962*I          71715.22613+46446.52248*I
  8   -1.008231080+7.375296026*I        11665.91277+43192.99867*I
  9   -1.963172287+6.078976680*I        -8896.934846+20828.33220*I
 10   -2.686055948+4.788728697*I        -10585.94122+5023.016968*I
 11   -3.117796387+3.514836954*I        -5840.703284-1849.390820*I
 12   -3.246879544+2.277647900*I        -1445.711653-2825.510292*I
 13   -3.066121013+1.142281977*I        592.0451140-1480.250475*I
 14   -2.589729022+.2454559118*I        699.9099194-227.5254054*I
 15   -1.993798460-.3004495470*I        278.9230260+148.7383102*I
 16   -1.465189193-.5860994240*I        48.59173860+129.0569199*I
 17   -1.029279950-.7326743521*I        -19.32486878+58.13012576*I
 18   -.6791822718-.7955247526*I        -21.56432216+15.52475319*I
 19   -.4200444206-.8151409714*I        -10.31232274+1.07336946*I
 20   -.2628441669-.8188672719*I        -2.717509426-.77954596*I
 21   -.2054850128-.8154652429*I        -.249920632-.17544905*I
 22   -.1987942922-.8133926184*I        -.2373433e-2-.437889e-2*I
 23   -.1987095325-.8133126080*I        .772e-6-.249e-5*I
 24   -.1987095315-.8133125470*I        -.6e-8+0.*I

The approximate solution is r4 = -.1987095315-.8133125470*I
with f(r4) = -.6e-8+0.*I
```

$r4 := -0.1987095315 - 0.8133125470\,I$

Success! For the fifth root, we again take the conjugate of the fourth root.

```
> r5:=conjugate(r4);
```

$r5 := -0.1987095315 + 0.8133125470\,I$

Just to show you can find complex roots with real first approximations, we try to find *r4* with real first approximations.

```
> r4:='r4';
```

$r4 := r4$

```
> r4:=muller(p,15,8,7,.000001,100,r4);
  i    p                                 f(p)
  -    -                                 ----
  0   15.                               1242805.
  1   8.                                66259.
  2   7.                                35373.
  3   6.599481019+1.379228196*I         16802.31801+24684.15470*I
  4   5.075302057+1.863606289*I         -1309.259618+10695.83422*I
  5   4.107132832+2.122431896*I         -3701.703837+3918.440216*I
  6   3.211248017+2.299523367*I         -2830.507107+230.9626133*I
  7   2.363040583+2.262570547*I         -1121.822840-832.5632882*I
  8   1.656550467+2.126141222*I         -180.5890381-696.3494286*I
  9   1.056104219+1.924097865*I         142.2748077-339.0553755*I
 10   .5660844358+1.680073001*I         151.5500584-94.29066120*I
 11   .1911312968+1.430017187*I         80.48561341+3.39048600*I
 12   -.716617336e-1+1.196478949*I      25.64293673+19.91680025*I
 13   -.2217551448+.9955330880*I        2.092097092+10.79882062*I
 14   -.2547001403+.8401193417*I        -2.235174976+1.74152988*I
 15   -.1960008272+.8071440202*I        .26214102e-1-.28288215*I
 16   -.1987970096+.8133414796*I        -.3164674e-2+.233930e-2*I
 17   -.1987095620+.8133125147*I        -.1673e-5-.90e-6*I
 18   -.1987095314+.8133125469*I        0.+.1e-7*I
```

```
The approximate solution is r4 = -.1987095314+.8133125469*I
with f(r4) = 0.+.1e-7*I
```

$$r4 := -0.1987095314 + 0.8133125469\ I$$

Success!