MATRIX REGULAR OPERATOR SPACES

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Abstract. The concept of the regular (or Riesz) norm on ordered real Banach spaces is generalized to matrix ordered complex operator spaces in a way that respects the matricial structure of the operator space. A norm on an ordered real Banach space E is regular if: (1) $-x \leq y \leq x$ implies that $\|y\| \leq \|x\|$; and (2) $\|y\| < 1$ implies the existence of $x \in E$ such that $\|x\| < 1$ and $-x \leq y \leq x$. A matrix ordered operator space is called matrix regular if, at each matrix level, the restriction of the norm to the self-adjoint elements is a regular norm. In such a space, elements at each matrix level can be written as linear combinations of four positive elements. The concept of the matrix ordered operator space is made specific in such a way as to be a natural generalization of ordered real and complex Banach spaces. For the case where V is a matrix ordered operator space, a natural cone is defined on the operator space $X^* \otimes^h V \otimes^h X$, with $\otimes^h$ indicating the Haagerup tensor product, so as to make it a matrix ordered operator space. Exploiting the advantages gained by taking X to be the column Hilbert space H_c, an equivalence is established between the matrix regularity of a space and that of its operator dual. This concept of matrix regularity also provides for more accessible proofs of the Christensen-Sinclair Multilinear Representation and Multilinear Decomposition theorems.