Cross Product for Vectors in 3-Dimensions

```
> restart:with(plots):with(plottools):with(VectorCalculus)
  :BasisFormat(false):
> setoptions3d(axes=NORMAL,labels=["x","y","z"],orientation=[20,70]
  );
```

The cross product is computed the same way in both the VectorCalculus and LinearAlgebra packages. We enter two vectors abstractly, with variable components.

```
> V:=Vector([v[1],v[2],v[3]]);W:=Vector([w[1],w[2],w[3]]);
```

$$V := \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix}$$

$$W := \begin{bmatrix} w_1 \\ w_2 \\ w_3 \end{bmatrix}$$

To find the **cross product** **V**x**W** of two vectors in three dimensional space, which is a vector, we use the CrossProduct command from the NumericalAnalysis package.

```
> CrossProduct(V,W);
```

$$\begin{bmatrix} v_2 w_3 - v_3 w_2 \\ v_3 w_1 - v_1 w_3 \\ v_1 w_2 - v_2 w_1 \end{bmatrix}$$

We can also use &x as an abbreviation of the above command.

```
> V &x W;
```

$$\begin{bmatrix} v_2 w_3 - v_3 w_2 \\ v_3 w_1 - v_1 w_3 \\ v_1 w_2 - v_2 w_1 \end{bmatrix}$$

Something interesting happens if we reverse the order of **V** and **W** and compute **W**x**V**.

```
> CrossProduct(W,V);
```

$$\begin{bmatrix} v_3 w_2 - v_2 w_3 \\ v_1 w_3 - v_3 w_1 \\ v_2 w_1 - v_1 w_2 \end{bmatrix}$$

We get a vector opposite the one above, i.e., **W**x**V** = **-(V**x**W)**, so this multiplication is **anti-commutative** as opposed to our usual experience of commutative multiplication.

Now let's look at a particular example.

```
> v:=Vector([1,2,3]);w:=Vector([5,-2,1]);
```

$$v := \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

$$w := \begin{bmatrix} 5 \\ -2 \\ 1 \end{bmatrix}$$

```
> vxw:=CrossProduct(v,w);
```

$$vxw := \begin{bmatrix} 8 \\ 14 \\ -12 \end{bmatrix}$$

We find the dot product of the cross product with each of the two original vectors.

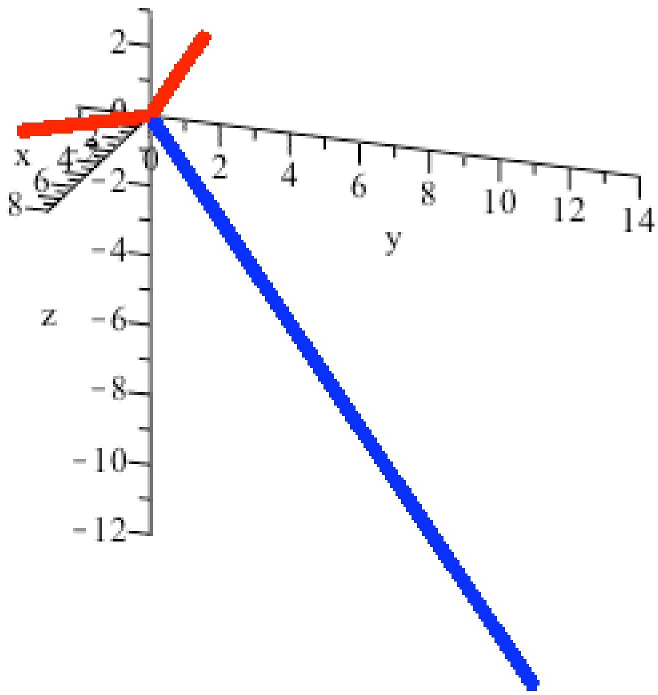
```
> vxw_dot_v:=DotProduct(vxw,v);vxw_dot_w:=DotProduct(vxw,w);
```

$$vxw\_dot\_v := 0$$

$$vxw\_dot\_w := 0$$

This means that the cross product is orthogonal or perpendicular to each of the original vectors. We plot the original vectors (red) and the cross product (blue).

```
> p1:=PlotVector(v,width=.4,head_width=.2,head_length=.2,scaling=
  constrained,color=red):
  p2:=PlotVector(w,width=.4,head_width=.2,head_length=.2,scaling=
  constrained,color=red):
  p3:=PlotVector(vxw,width=.4,head_width=.2,head_length=.2,scaling=
  constrained,color=blue):
  display(p1,p2,p3);
```



We do see that the cross product is perpendicular to the original vectors.