Level Sets for Functions of Three Variables

```
> restart:with(plots):
> setoptions3d(axes=NORMAL,labels=["x","y","z"],orientation=[20,
  70]);
```

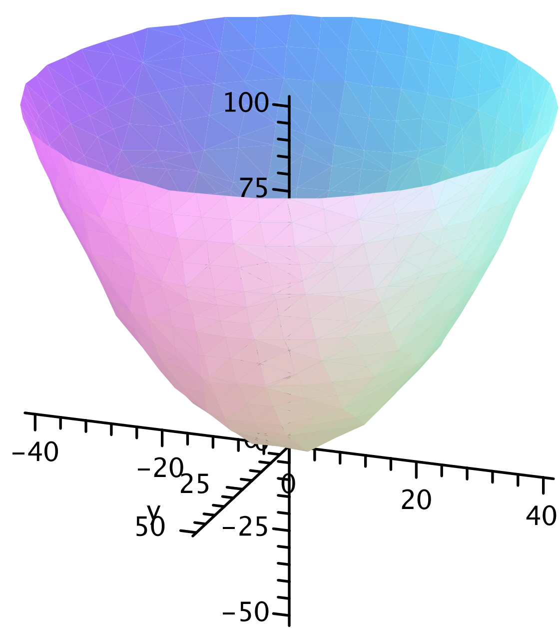
Functions of three variables cannot be graphed in the usual sense since four dimensions would be required. What can be done to help understand the function is to graph various level sets, which are graphs in three dimensions. We begin with a function of the form $f(x, y, z) = \frac{x^2}{a^2} + \frac{y^2}{b^2} - z$.

```
> f:=x^2/25+y^2/16-z;
```

$$f := \frac{1}{25}x^2 + \frac{1}{16}y^2 - z$$

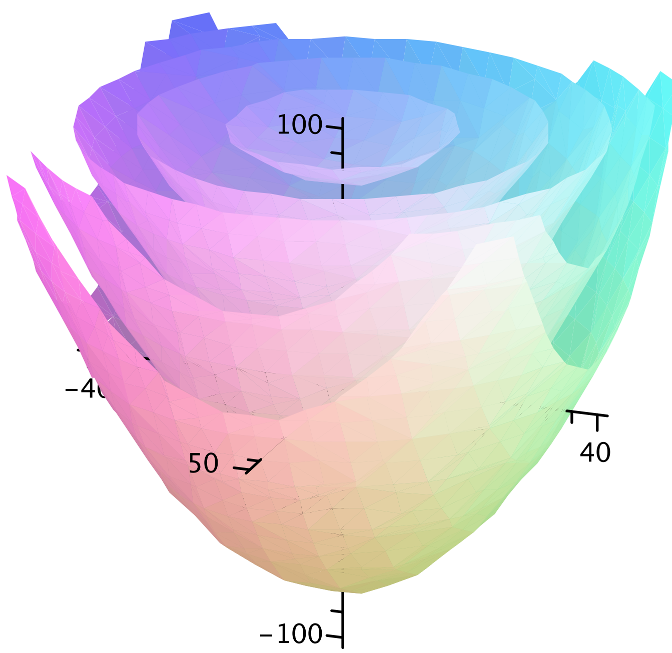
Level sets are of the form $\frac{x^2}{25} + \frac{y^2}{16} - z = c$ or $\frac{x^2}{25} + \frac{y^2}{16} - z - c = 0$. Let's look at c=0.

```
> implicitplot3d(x^2/25+y^2/16-z=0,x=-50..50,y=-40..40,z=-50..100,
  style=patchnogrid);
```



This surface is an **elliptical parabola** since horizontal cross sections are ellipses and vertical cross sections perpendicalar to the x-and y-axes are parabolas. It is also the graph of the function $f(x, y) = \frac{x^2}{25} + \frac{y^2}{16}$. In general, the surface which is the graph of a function $z = f(x, y)$ is also the level surface $g(x, y, z) = 0$ for the function $g(x, y, z) = f(x, y) - z$. To aid in understanding our original function $f(x, y, z) = \frac{x^2}{25} + \frac{y^2}{16} - z$, we superimpose the plots for levels c=-80, -40, 0, 40, and 80.

```
> p1:=implicitplot3d(x^2/25+y^2/16-z=-80,x=-50..50,y=-40..40,z=
  -100..100,style=patchnogrid):
  p2:=implicitplot3d(x^2/25+y^2/16-z=-40,x=-50..50,y=-40..40,z=
  -100..100,style=patchnogrid):
  p3:=implicitplot3d(x^2/25+y^2/16-z=0,x=-50..50,y=-40..40,z=-100.
  .100,style=patchnogrid):
  p4:=implicitplot3d(x^2/25+y^2/16-z=40,x=-50..50,y=-40..40,z=
  -100..100,style=patchnogrid):
  p5:=implicitplot3d(x^2/25+y^2/16-z=80,x=-50..50,y=-40..40,z=
  -100..100,style=patchnogrid):
  display(p1,p2,p3,p4,p5);
```



In this case we see that we get a layered effect from translations of the original graph. In the equation, $f(x, y, z) = c$, we can think of c as a time variable that gives us a different elliptic paraboloid at each moment of time.