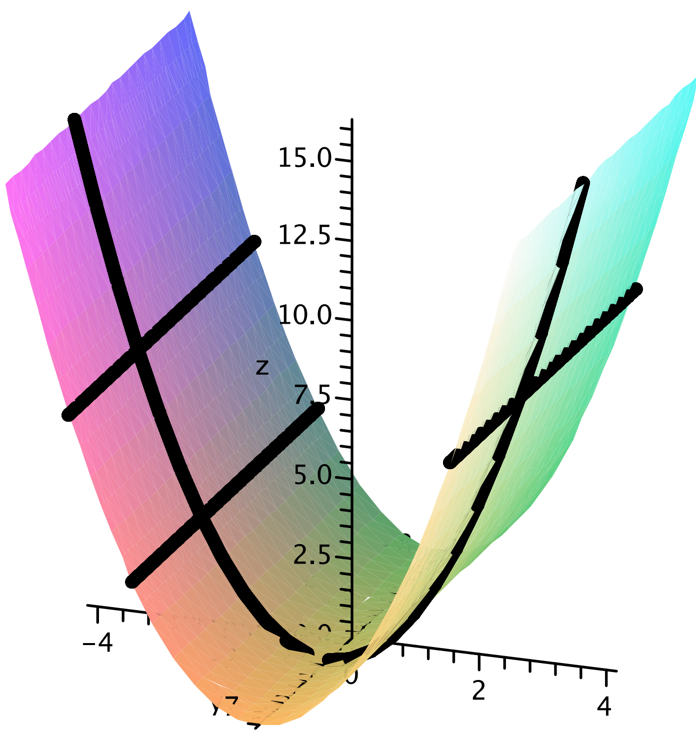
Surfaces in Space

```
> restart:with(plots):
> setoptions3d(axes=NORMAL,labels=["x","y","z"],orientation=[20,
  70]);
```

Cylinders

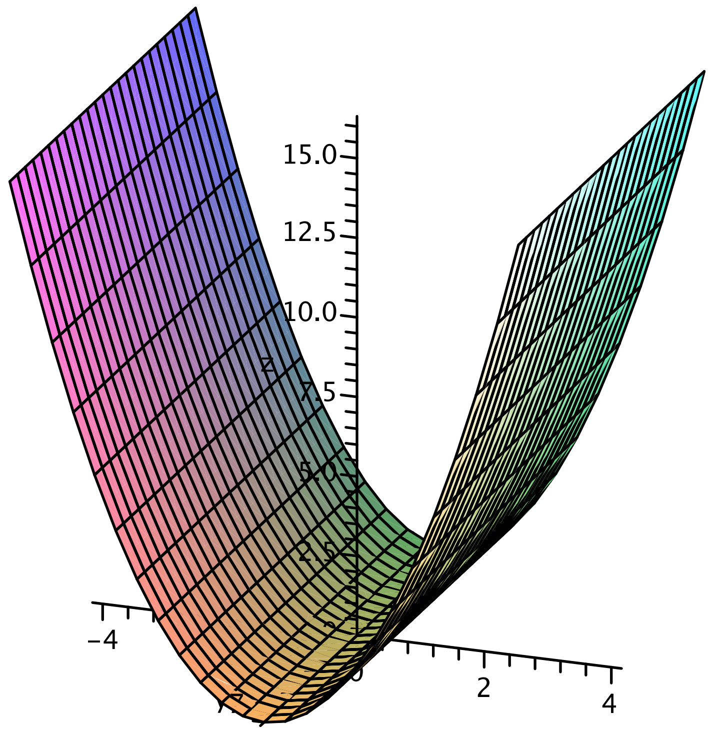
You get a surface called a **cylinder** by taking a space curve, such as a parabola in the example below, and a line intersecting the curve, and then moving the line along the curve in parallel fashion to sweep out a surface. Then the traces in any plane parallel to a given plane are the same

```
> p1:=plot3d([x,y,y^2],x=-8..8,y=-4..4,style=patchnogrid):
  p2:=plot3d([x,-3,9],x=-8..8,y=-4..4, thickness=5):
  p3:=plot3d([x,-2,4],x=-8..8,y=-4..4, thickness=5):
  p4:=plot3d([x,3,9],x=-8..8,y=-4..4, color=blue, thickness=5):
  p5:=plot3d([2,y,y^2],x=-8..8,y=-4..4, thickness=5):
  display(p1,p2,p3,p4,p5);
```



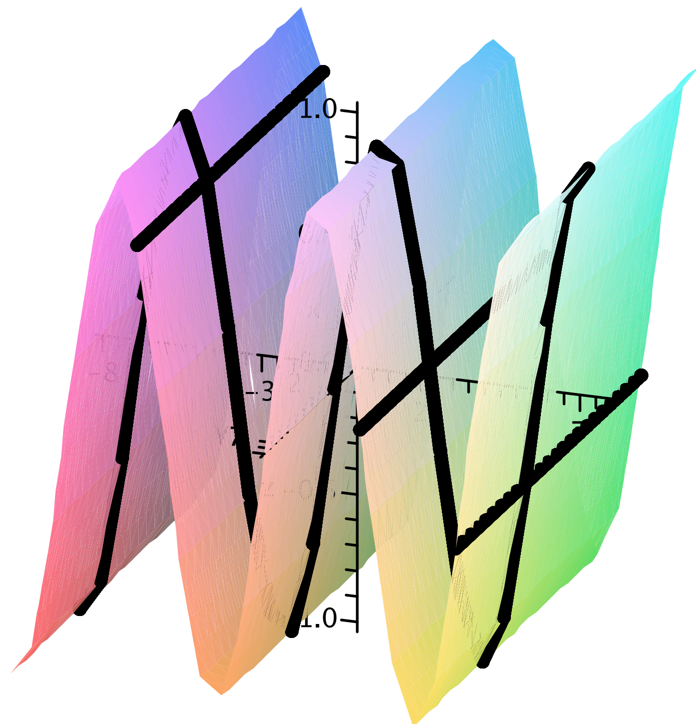
The above is the graph of $z = y^2$, which in three-dimensional space needs to be considered as $z = 0\,x + y^2$. A cylinder is a type of "ruled surface." You can clearly see the parallel lines by redoing the plot with the grid option enabled. Notice the use of parametric plotting in the five graphs above, with *x* and *y* as parameters.

```
> plot3d([x,y,y^2],x=-8..8,y=-4..4);
```



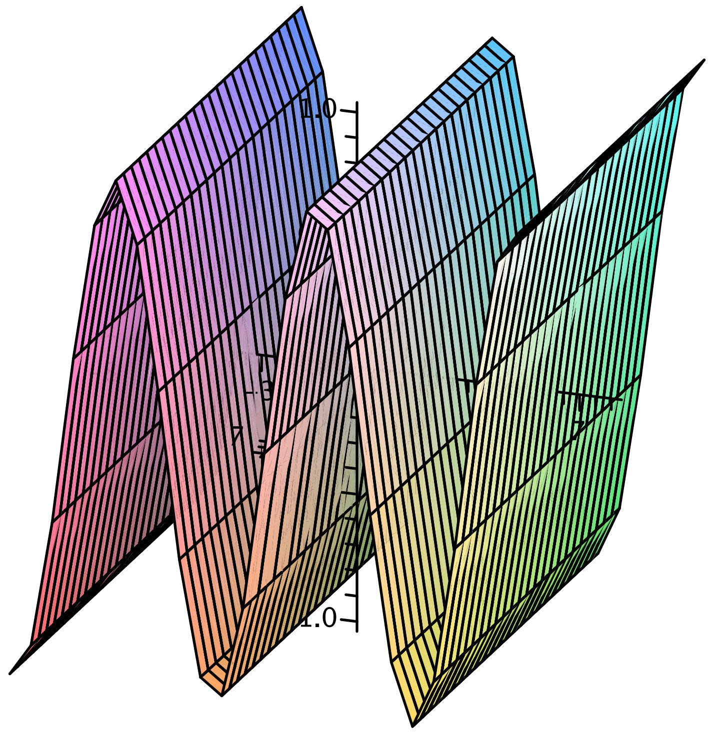
Replacing the parabola with a sine curve gives an interesting cylinder.

```
> p1:=plot3d([x,y,sin(y)],x=-8..8,y=-8..8,style=patchnogrid):
  p2:=plot3d([x,-4,sin(-4)],x=-8..8,y=-8..8, thickness=5):
  p3:=plot3d([x,6,sin(6)],x=-8..8,y=-8..8, thickness=5):
  p4:=plot3d([x,3,sin(3)],x=-8..8,y=-8..8, color=blue, thickness=
  5):
  p5:=plot3d([2,y,sin(y)],x=-8..8,y=-8..8, thickness=5):
  display(p1,p2,p3,p4,p5);
```



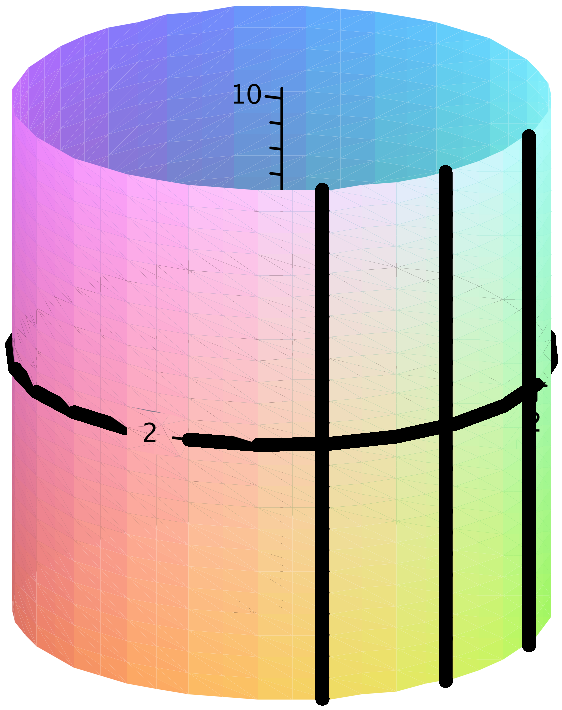
This cylinder is the graph of $z = \sin(y)$ in three dimensional space. Again, look at it with the grid option.

```
> plot3d([x,y,sin(y)],x=-8..8,y=-8..8);
```



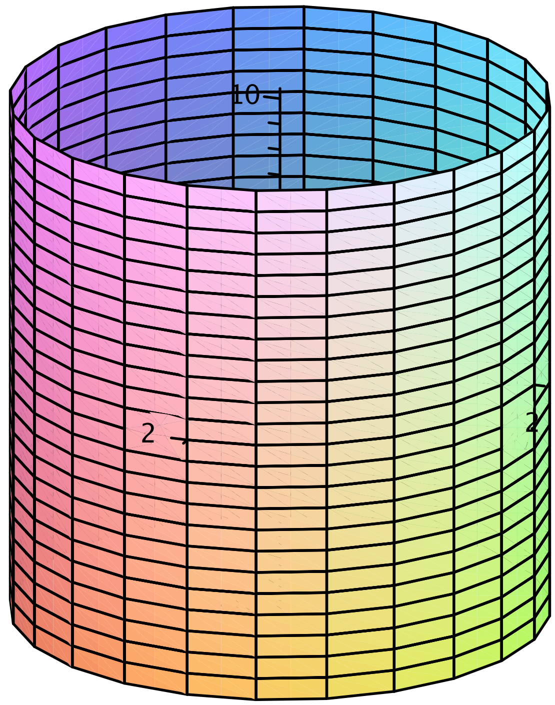
Finally, let's consider the **right circular cylinder**, what we usually think of as a cylinder, with equation $x^2 + y^2 = 4$. Notice the parametric plots used here with x and z as the parameters.

```
> p1:=plot3d([2*cos(x),2*sin(x),z],x=0..2*Pi,z=-10..10,style=
  patchnogrid):
  p2:=plot3d([2*cos(1),2*sin(1),z],x=0..2*Pi,z=-10..10, thickness=
  5):
  p3:=plot3d([2*cos(.5),2*sin(.5),z],x=0..2*Pi,z=-10..10,
  thickness=5):
  p4:=plot3d([2*cos(1.5),2*sin(1.5),z],x=0..2*Pi,z=-10..10, color=
  blue, thickness=5):
  p5:=plot3d([2*cos(x),2*sin(x),0],x=0..2*Pi,z=-10..10, thickness=
  5):
  display(p1,p2,p3,p4,p5);
```



Finally, we view the right circular cylinder with the grid option.

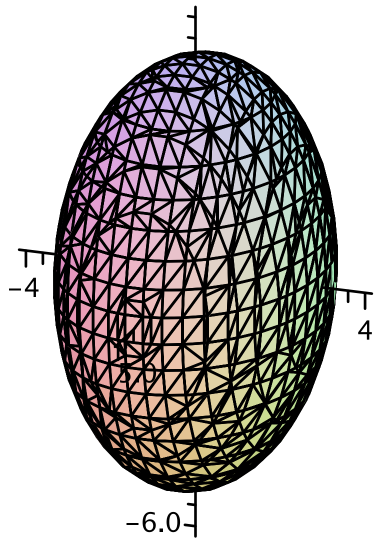
```
> plot3d([2*cos(x),2*sin(x),z],x=0..2*Pi,z=-10..10);
```



Quadric Surfaces

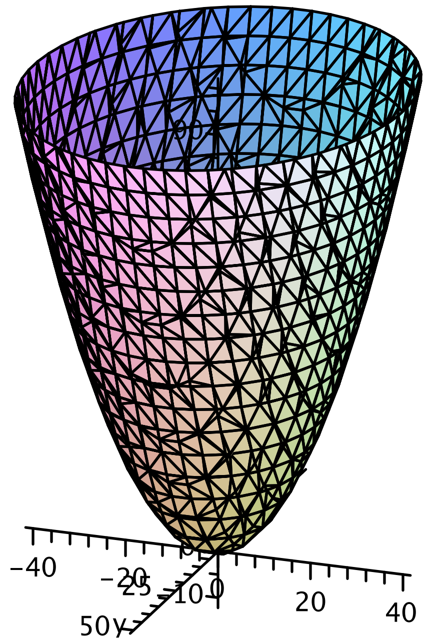
We will use implicitplot3d for graphs in three dimensions. We begin with an **ellipsoid** of the form $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$, namely $\frac{x^2}{16}+\frac{y^2}{9}+\frac{z^2}{25}=1$.

```
> implicitplot3d(x^2/16+y^2/9+z^2/25=1,x=-5..5,y=-4..4,z=-6..6,
  scaling=constrained,numpoints=5000);
```



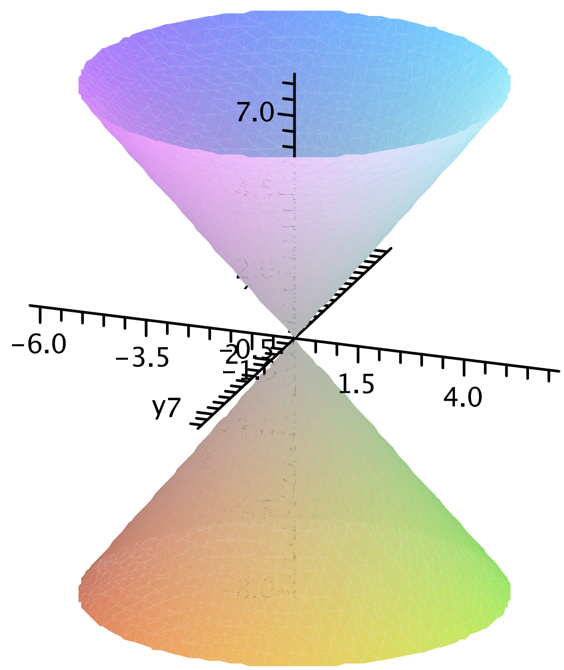
Cross sections perpendicular to each of the axes are ellipses. We get a **sphere** of radius r when $a^2 = b^2 = c^2 = r^2$. We next look at $z = \frac{x^2}{a^2} + \frac{y^2}{b^2} - c$, the graph of the function $f(x, y) = \frac{x^2}{a^2} + \frac{y^2}{b^2} - c$ We will take c=0 and look at $z = \frac{x^2}{25} + \frac{y^2}{16}$.

```
> implicitplot3d(z=x^2/25+y^2/16,x=-50..50,y=-40..40,z=-10..100,
  scaling=constrained,numpoints=5000);
```



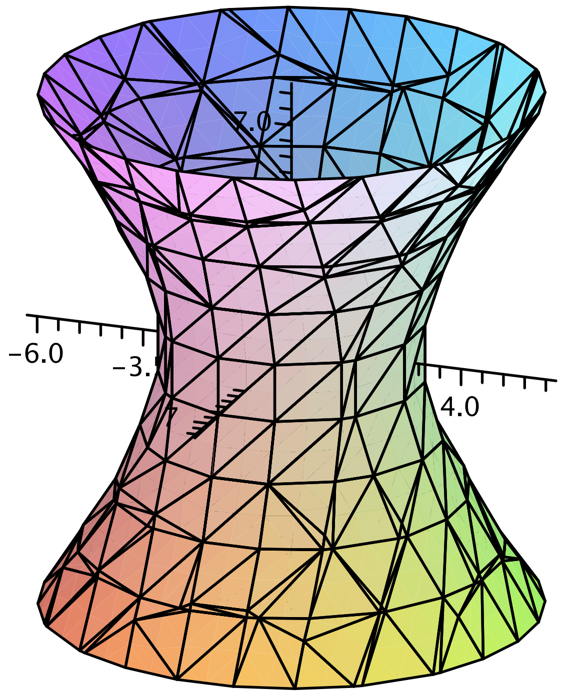
This surface is an **elliptic paraboloid** since horizontal cross sections are ellipses and vertical cross sections perpendicalar to the x-and y-axes are parabolas. If $a^2 = b^2$, you get a circular paraboloid. It is also the graph of the function $f(x, y) = \frac{x^2}{25} + \frac{y^2}{16}$. Next is the **elliptic cone** $\frac{x^2}{a^2} + \frac{y^2}{b^2} = \frac{z^2}{c^2}$. We consider $\frac{x^2}{16} + \frac{y^2}{9} = \frac{z^2}{25}$.

```
> implicitplot3d(x^2/16+y^2/9=z^2/25,x=-8..8,y=-6..6,z=-8..8,
  style=patchnogrid,numpoints=100000);
```



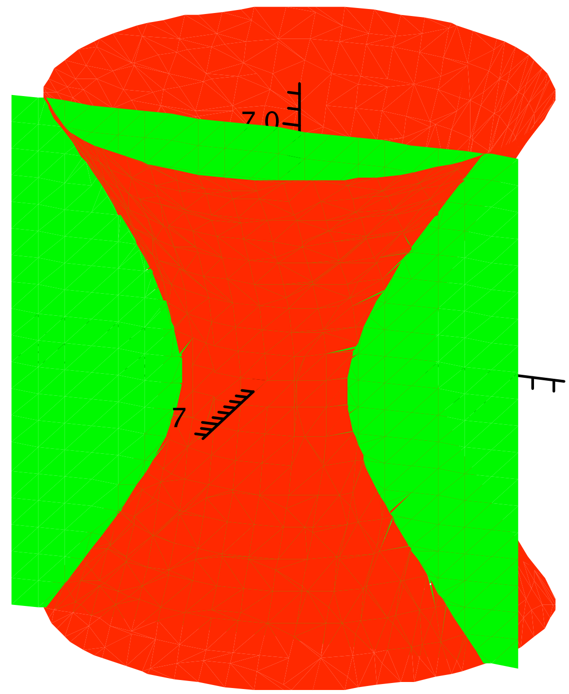
Horizontal cross sections are ellipses and vertical cross sections perpendicular to an axis are hyperbolas, unless they pass through the origin, in which case they are intersecting lines (degenerate hyperbolas). The large value of numpoints is needed to bring the two halves togeter in Maple. Next is the **hyperboloid of one sheet** $\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} = 1$. We look at $\frac{x^2}{16} + \frac{y^2}{9} - \frac{z^2}{25} = 1$.

```
> implicitplot3d(x^2/16+y^2/9-z^2/25=1,x=-8..8,y=-6..6,z=-8..8);
```



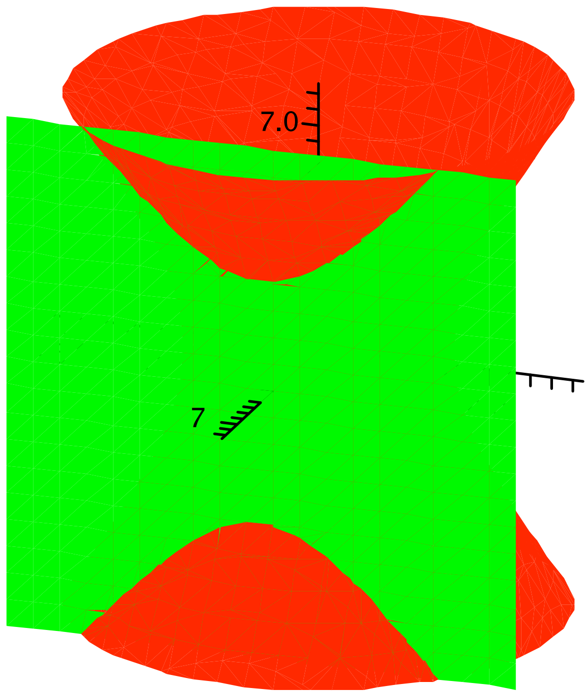
Horizontal cross sections are ellipses and vertical cross sections perpendicular to an axis are hyperbolas. Let's take a cross section perpendicular to the x-axis with $x < \sqrt{16}$.

```
> p1:=implicitplot3d(x^2/16+y^2/9-z^2/25=1,x=-8..8,y=-6..6,z=-8.
  .8,color=red,style=patchnogrid,numpoints=5000):
  p2:=implicitplot3d(x=3,x=-8..8,y=-6..6,z=-8..8,color=green,
  style=patchnogrid,numpoints=5000):
  display(p1,p2);
```



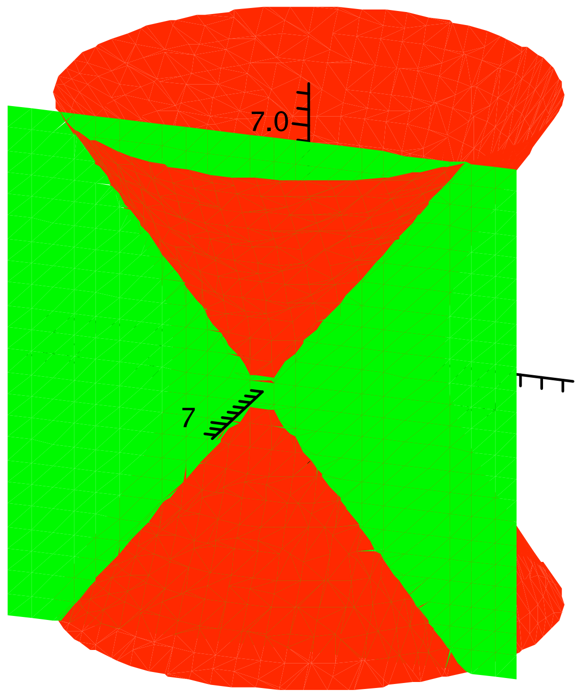
We get a hyperbola opening left-right. Now take a cross section perpendicular to the x-axis with $x > \sqrt{16}$.

```
> p1:=implicitplot3d(x^2/16+y^2/9-z^2/25=1,x=-8..8,y=-6..6,z=-8.
  .8,color=red,style=patchnogrid,numpoints=5000):
  p2:=implicitplot3d(x=5,x=-8..8,y=-6..6,z=-8..8,color=green,
  style=patchnogrid,numpoints=5000):
  display(p1,p2);
```



We get a hyperbola opening up-down. Finally, take a cross section perpendicular to the x-axis with $x = \sqrt{16}$.

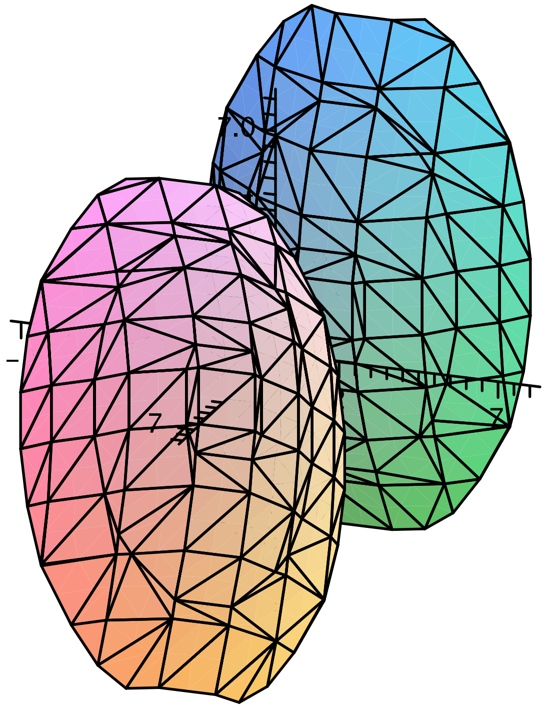
```
> p1:=implicitplot3d(x^2/16+y^2/9-z^2/25=1,x=-8..8,y=-6..6,z=-8.
  .8,color=red,style=patchnogrid,numpoints=10000):
  p2:=implicitplot3d(x=4,x=-8..8,y=-6..6,z=-8..8,color=green,
  style=patchnogrid,numpoints=10000):
  display(p1,p2);
```



We get (or would get if I set numpoints higher) two intersecting lines, which is a degenerate hyperbola.

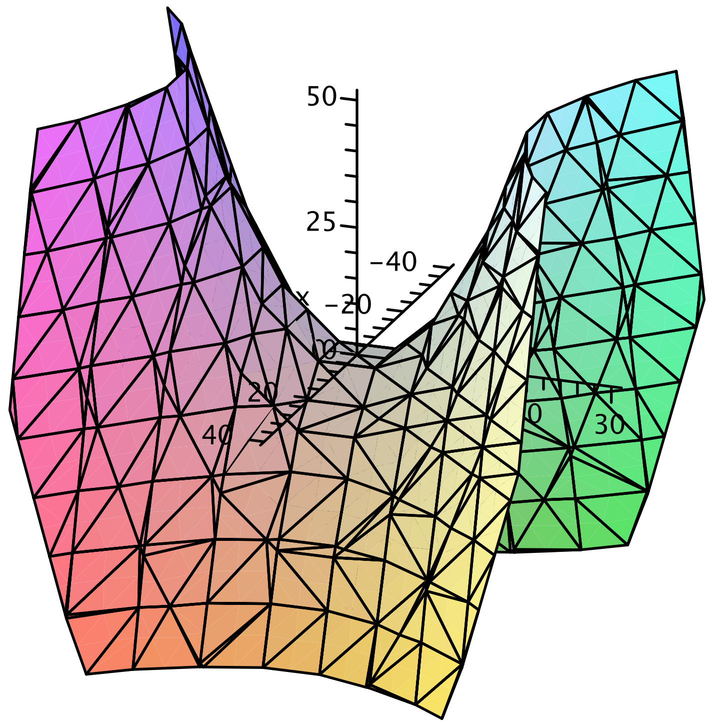
Next is the **hyperboloid of two sheets** $\frac{x^2}{a^2} - \frac{y^2}{b^2} - \frac{z^2}{c^2} = 1$. We look at $\frac{x^2}{16} - \frac{y^2}{9} - \frac{z^2}{25} = 1$.

```
> implicitplot3d(x^2/16-y^2/9-z^2/25=1,x=-8..8,y=-8..8,z=-8..8);
```



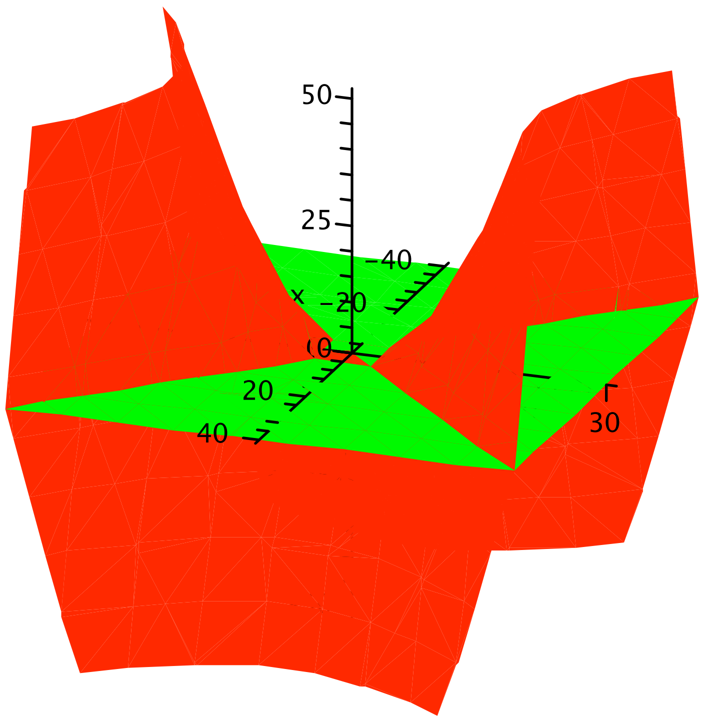
Cross sections perpendicular to the x-axis are ellipses while those perpendicular to the y and z-axes are hyperbolas. The final figure we wish to look at is the **hyperbolic paraboloid** (**saddle**) for functions of the form $z = -\frac{x^2}{a^2} + \frac{y^2}{b^2} + c$. We choose $z = -\frac{x^2}{16} + \frac{y^2}{9}$.

```
> implicitplot3d(z=-x^2/16+y^2/9,x=-40..40,y=-30..30,z=-50..50);
```



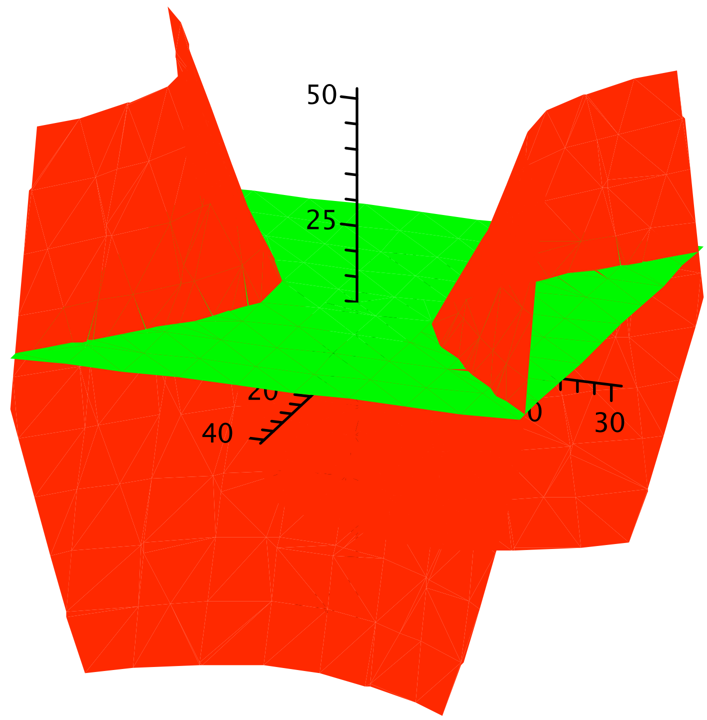
A horizonal plane through the origin meets the saddle in two intersecting lines.

```
> p1:=implicitplot3d(z=-x^2/16+y^2/9,x=-40..40,y=-30..30,z=-50.
  .50,style=patchnogrid,color=red):
  p2:=implicitplot3d(z=0,x=-40..40,y=-30..30,z=-50..50,style=
  patchnogrid,color=green):
  display(p1,p2);
```



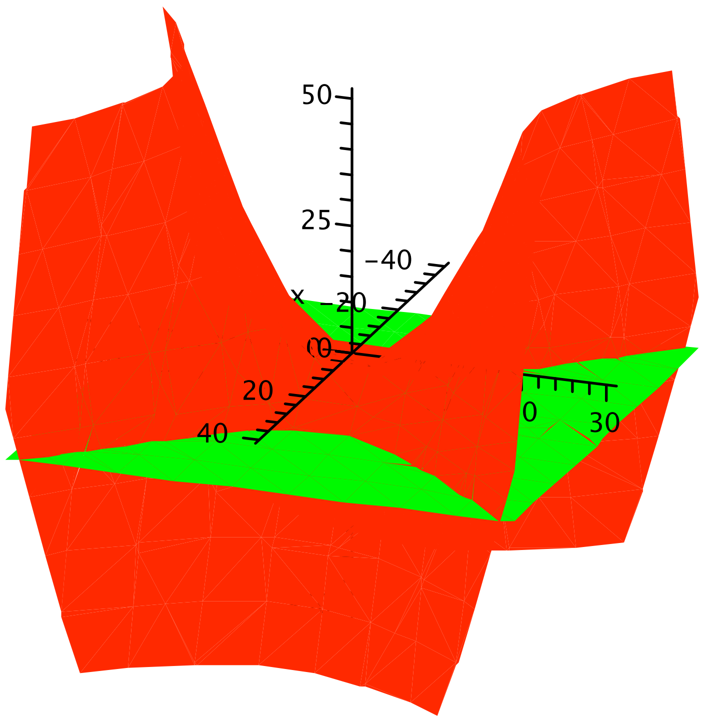
A horizonal plane with $0 < z$ meets the saddle in a hyperbola opening along the y-axis.

```
> p1:=implicitplot3d(z=-x^2/16+y^2/9,x=-40..40,y=-30..30,z=-50.
  .50,style=patchnogrid,color=red):
  p2:=implicitplot3d(z=10,x=-40..40,y=-30..30,z=-50..50,style=
  patchnogrid,color=green):
  display(p1,p2);
```



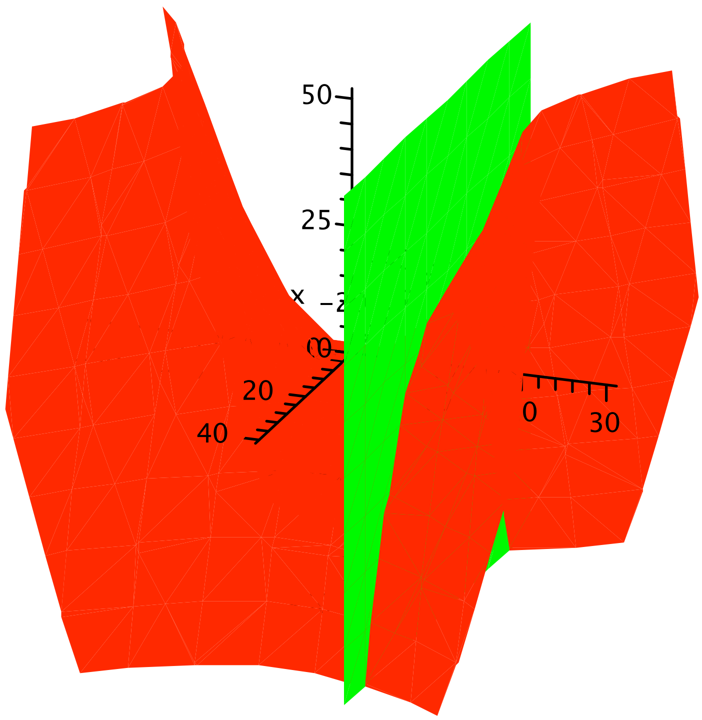
A horizonal plane with $0 > z$ meets the saddle in a hyperbola opening along the x-axis.

```
> p1:=implicitplot3d(z=-x^2/16+y^2/9,x=-40..40,y=-30..30,z=-50.
  .50,style=patchnogrid,color=red):
  p2:=implicitplot3d(z=-10,x=-40..40,y=-30..30,z=-50..50,style=
  patchnogrid,color=green):
  display(p1,p2);
```



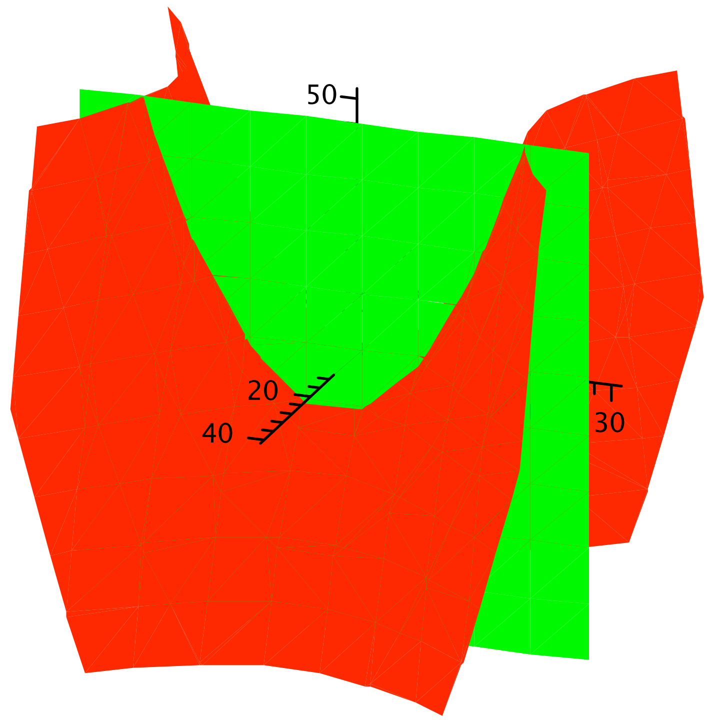
A vertical plane perpendicular to the y-axis meets the saddle in a parabola opening downward.

```
> p1:=implicitplot3d(z=-x^2/16+y^2/9,x=-40..40,y=-30..30,z=-50.
  .50,style=patchnogrid,color=red):
  p2:=implicitplot3d(y=10,x=-40..40,y=-30..30,z=-50..50,style=
  patchnogrid,color=green):
  display(p1,p2);
```



A vertical plane perpendicular to the x-axis meets the saddle in a parabola opening upward.

```
> p1:=implicitplot3d(z=-x^2/16+y^2/9,x=-40..40,y=-30..30,z=-50.
  .50,style=patchnogrid,color=red):
  p2:=implicitplot3d(x=10,x=-40..40,y=-30..30,z=-50..50,style=
  patchnogrid,color=green):
  display(p1,p2);
```


50
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40
30