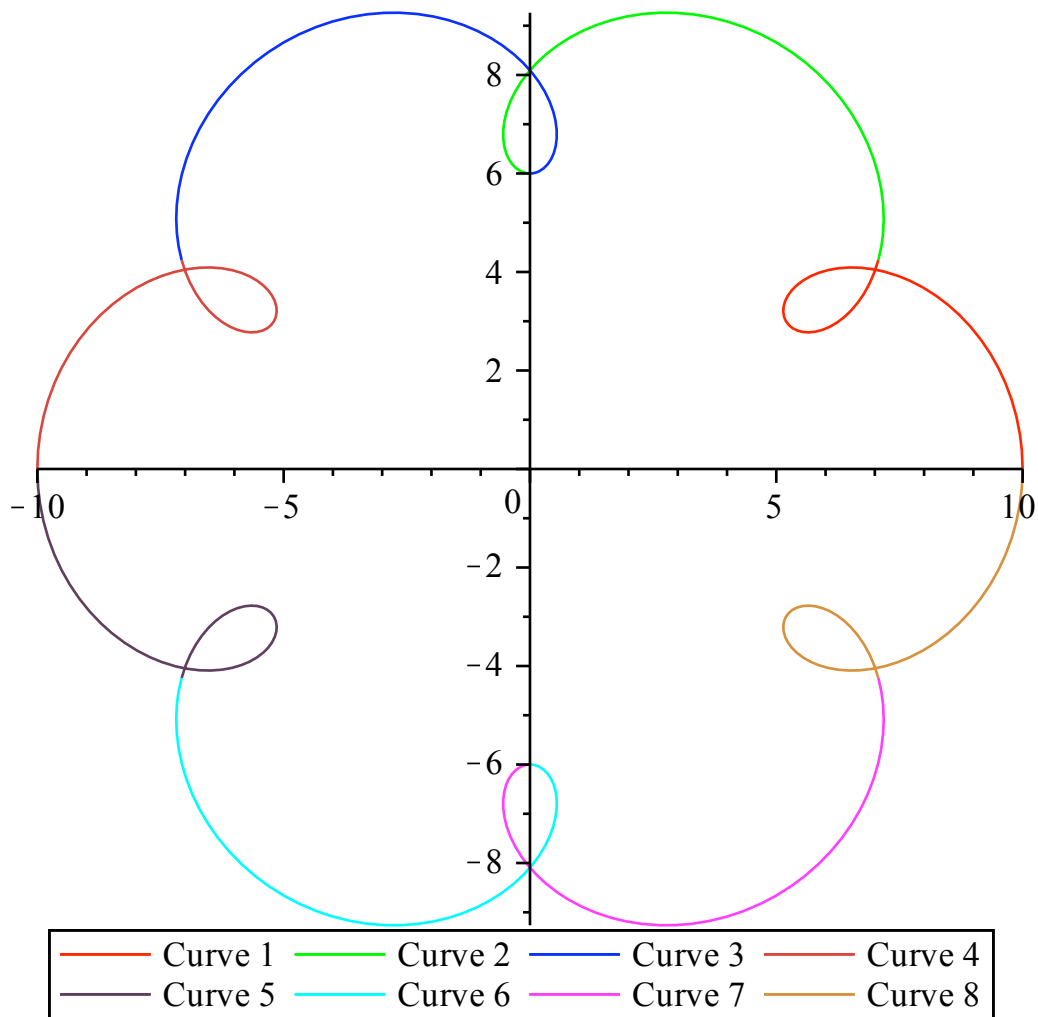


## Parameterized Curves

```
> restart:with(plots):with(VectorCalculus):  
> setoptions3d(axes=NORMAL,labels=["x","y","z"],orientation=[20,70]  
):
```

We first graph the curve parameterized by  $x = 8 \cos(t) + 2 \cos(7t)$ ,  $y = 8 \sin(t) + 2 \sin(7t)$ . But we graph the curve in eight stages, noting  $t=0..2\pi$  produces the entire curve.

```
> p1:=plot([8*cos(t)+2*cos(7*t),8*sin(t)+2*sin(7*t),t=0..Pi/4],  
scaling=constrained,color=red):  
p2:=plot([8*cos(t)+2*cos(7*t),8*sin(t)+2*sin(7*t),t=Pi/4..Pi/2],  
scaling=constrained,color=green):  
p3:=plot([8*cos(t)+2*cos(7*t),8*sin(t)+2*sin(7*t),t=Pi/2..3*  
Pi/4],scaling=constrained,color=blue):  
p4:=plot([8*cos(t)+2*cos(7*t),8*sin(t)+2*sin(7*t),t=3*Pi/4..Pi],  
scaling=constrained,color=orange):  
p5:=plot([8*cos(t)+2*cos(7*t),8*sin(t)+2*sin(7*t),t=Pi..5*Pi/4],  
scaling=constrained,color=violet):  
p6:=plot([8*cos(t)+2*cos(7*t),8*sin(t)+2*sin(7*t),t=5*Pi/4..3*  
Pi/2],scaling=constrained,color=cyan):  
p7:=plot([8*cos(t)+2*cos(7*t),8*sin(t)+2*sin(7*t),t=3*Pi/2..7*  
Pi/4],scaling=constrained,color=magenta):  
p8:=plot([8*cos(t)+2*cos(7*t),8*sin(t)+2*sin(7*t),t=7*Pi/4..2*  
Pi],scaling=constrained,color=gold):  
display(p1,p2,p3,p4,p5,p6,p7,p8);
```



It is clear that the curve is being plotted in a counterclockwise direction. Let's find the length of this curve.

```
> f:=8*cos(t)+2*cos(7*t):
g:=8*sin(t)+2*sin(7*t):
s:=int(sqrt((diff(f,t))^2+(diff(g,t))^2),t=0..2*Pi);
s_approx:=evalf(s);
```

$$s := 24 \operatorname{EllipticE}\left(\frac{4}{3} I \sqrt{7}\right)$$

$$s\_approx := 95.30571353 - 0. I$$

Most of the integrals that we get for finding arclength are very difficult (or impossible) to evaluate using the Fundamental Theorem of Calculus. So we turn to a numerical method such as **Simpson's rule**. We can also find the arc length by using the [ArcLength](#) command from the [VectorCalculus](#) package.

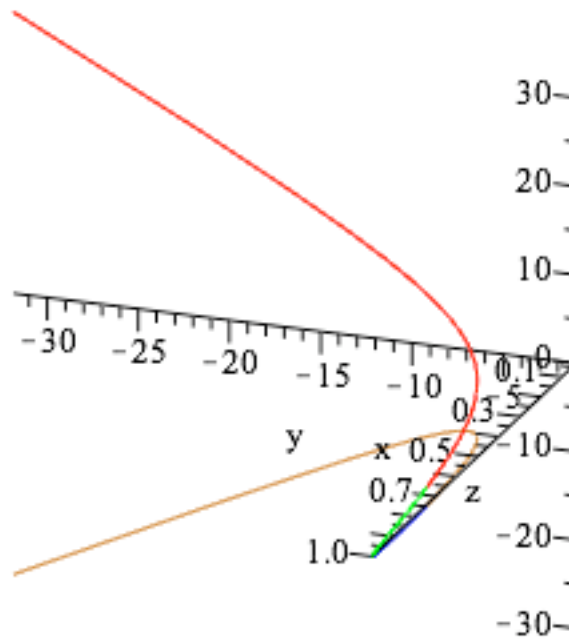
```
> sapprox:=ArcLength(<f,g>,t=0..2*Pi);
```

$$sapprox := 24 \operatorname{EllipticE}\left(\frac{4}{3} I \sqrt{7}\right)$$

Now we move to three dimensions and plot the curve given parametrically by the equations  $x = \sin(t)$ ,  $y = -\csc(t)$ ,  $z = \cot$ . We again break the formation of the plot into parts, but from  $t = \frac{\pi}{100}$  to  $t = \frac{99\pi}{100}$ .

Why not  $t = 0 \dots \pi$  you might ask?

```
> p1:=spacecurve([sin(t),-csc(t),cot(t)],t=Pi/100..Pi/4,color=red):  
p2:=spacecurve([sin(t),-csc(t),cot(t)],t=Pi/4..Pi/2,color=green):  
p3:=spacecurve([sin(t),-csc(t),cot(t)],t=Pi/2..3*Pi/4,color=blue):  
:  
p4:=spacecurve([sin(t),-csc(t),cot(t)],t=3*Pi/4..99*Pi/100,color=  
gold):  
display(p1,p2,p3,p4);
```

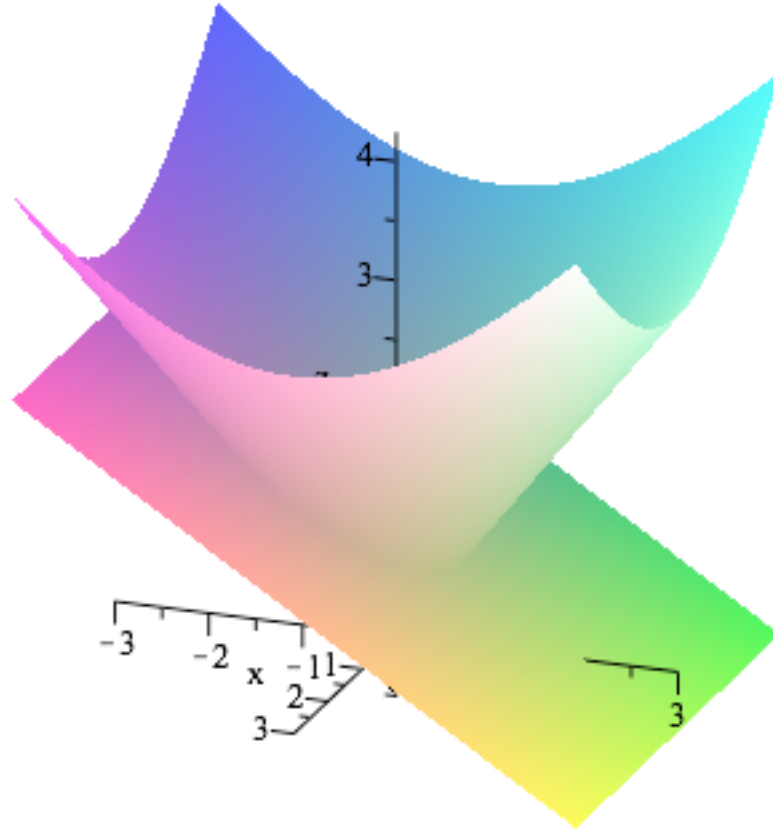


Let's find the length of this curve.

```
> f:=sin(t):  
g:=-csc(t):  
h:=cot(t):  
s:=evalf(ArcLength(<f,g,h>,t=Pi/100..99*Pi/100));  
s := 88.98543743
```

Next we look at the intersection of the cone  $z = \sqrt{x^2 + y^2}$  with the plane  $y + 2z = 2$ .

```
> p1:=plot3d(sqrt(x^2+y^2),x=-3..3,y=-3..3,style=patchnogrid):  
p2:=plot3d(1/2*(2-y),x=-3..3,y=-3..3,style=patchnogrid):  
display(p1,p2);
```



We equate the two expressions for  $z$  and solve for  $y$  in terms of  $x$ .

```
> y:=solve(sqrt(x^2+y^2)=1/2*(2-y),y);
```

$$y := -\frac{2}{3} - \frac{2}{3} \sqrt{4 - 3x^2}, -\frac{2}{3} + \frac{2}{3} \sqrt{4 - 3x^2}$$

We solve for  $z$  in terms of  $y$ .

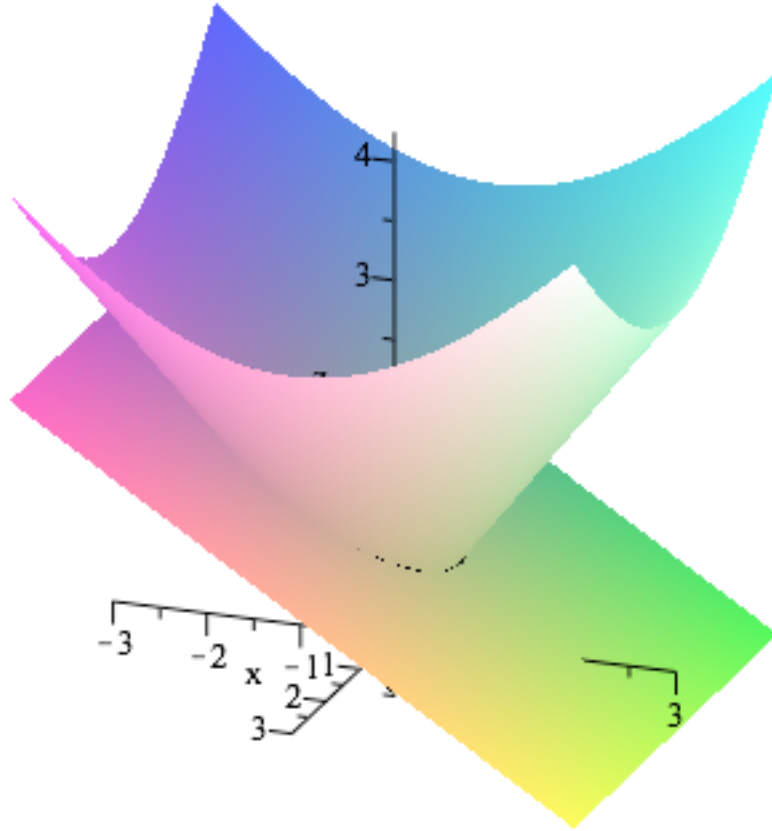
```
> z[1]:=1/2*(2-y[1]);z[2]:=1/2*(2-y[2]);
```

$$z_1 := \frac{4}{3} + \frac{1}{3} \sqrt{4 - 3x^2}$$

$$z_2 := \frac{4}{3} - \frac{1}{3} \sqrt{4 - 3x^2}$$

We add the two curves which together form an ellipse.

```
> p3:=spacecurve([t,-2/3-(2/3)*sqrt(4-3*t^2),4/3+(1/3)*sqrt(4-3*t^2)],t=-2/sqrt(3)..2/sqrt(3),color=black,thickness=2);
p4:=spacecurve([t,-2/3+(2/3)*sqrt(4-3*t^2),4/3-(1/3)*sqrt(4-3*t^2)],t=-2/sqrt(3)..2/sqrt(3),color=black,thickness=2);
display(p1,p2,p3,p4);
```



Finally, we find the length of the curve of intersection.

```

> f1:=t:
  g1:=-2/3-(2/3)*sqrt(4-3*t^2):
  h1:=4/3+(1/3)*sqrt(4-3*t^2):
  f2:=t:
  g2:=-2/3+(2/3)*sqrt(4-3*t^2):
  h2:=4/3-(1/3)*sqrt(4-3*t^2):
> s:=ArcLength(<f1,g1,h1>,t=-2/sqrt(3)..2/sqrt(3))+ArcLength(<f2,
  g2,h2>,t=-2/sqrt(3)..2/sqrt(3));
> s:=evalf(s);

```

$$s := \frac{8}{3} \sqrt{3} \operatorname{EllipticE}\left(\frac{1}{3} \operatorname{I} \sqrt{2} \sqrt{3}\right)$$

$$s := 8.344362537 - 0. \operatorname{I}$$