

## Maple Worksheets for Ordinary Differential Equations

Complimentary software to accompany the textbook:

**Differential Equations: Concepts, Methods, and Models (2001-2002 Edition)**

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### Getting to Know Maple

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Use the mouse to place the blinking cursor after the "greater than" symbol ( $>$ ), immediately after this paragraph, unless it is already there. The symbol,  $>$ , is the Maple *prompt*. Then hit ENTER after you have read this paragraph. To keep reading more text or to execute a command, simply hit ENTER as many times as necessary. Make sure that you execute all commands and in the order in which they appear; for otherwise you may get responses that will not make sense.

>

**INTRODUCING MAPLE WITH ARITHMETIC.** We begin this first worksheet with some commands to show you how to do arithmetic with Maple. To use one of the following arithmetic operations, use the symbol to its right.

- addition +
- subtraction -
- multiplication \*
- division /
- exponentiation ^ or \*\*
- factorial !
- absolute value abs()

>

The following are some Maple commands. Before you execute each of the following commands, predict how you think Maple will respond. Then execute the command by hitting ENTER to see the actual output that appears in the middle of the next line.

> 1-2;	-1
> 1-2+3;	2
> 1-(2+3);	-4
> 100-20/4;	95
> (100-20)/4;	20
> 100-20/4+2*3-1;	100
> 10/24;	$\frac{5}{12}$
> 10/24+2/3;	$\frac{13}{12}$

>

Note the order in which Maple carried out the previous operations. If you are not sure about the order of operations when writing Maple commands, use parenthesis.

>

>  $-2^4$ ;  
-16

>  $(-2)^4$ ;  
16

>  $\text{abs}(-2^4)$ ;  
16

>  $5!$ ;  
120

>  $10/12+1$ ;  
 $\frac{11}{6}$

>

Note that we have used a semicolon after each of the previous commands. In fact, every command in Maple must end with either a semicolon or a colon. The difference between these two endings is that the *semicolon* displays the output whereas the *colon* suppresses the output, even though the command is still carried out. For example, compare the responses to the following commands:

>  $1*2$ ;  
2

>  $2*3$ ;

>

We can refer to the previous product of 2 times 3 without having to type it again by using the *ditto operator*. It is represented by the percentage sign (%). To see that the multiplication has actually been carried out, we place a semicolon after the percentage sign.

> %;  
6

This particular text that you are now reading is a Maple *paragraph*. Note the facing-right bracket on the left that encloses this particular paragraph. This paragraph contains three sentences.

>

More than one command may appear in the same line after a Maple prompt; for example, the next Maple prompt has three commands after it.

>  $2+4/8+1$ ;  $1+2+3$ ;  $((2^2)^2)^2$ ;  
 $\frac{7}{2}$   
256

>

Or, we may simply type just one command after each Maple prompt:

>

>  $2+4/8+1$ ;  
>  $1+2+3$ ;  
>  $((2^2)^2)^2$ ;  
 $\frac{7}{2}$   
6  
256

>

An *execution group* is either one command or several commands grouped together by a bracket. An execution group is easy to recognize--one or more Maple prompts are enclosed by a facing-right bracket. Up to now, all of the previous execution groups contained only one command, except for the last two. They each contained three commands.

>

**PREDEFINED CONSTANTS.** Well-known mathematical constants are obtained by executing the following commands:

> **Pi; 2\*Pi/6;**

$$\pi$$
$$\frac{1}{3} \pi$$

> **exp(1); # The Euler's number of calculus,that is, 2.71828...**

e

Note that the number sign (#) allows a comment to be made on the same line as a Maple command; in other words, Maple does not echo the commentary after #.

> **I;**

I

I represents a complex number whose square is -1, as the following command verifies.

> **I^2;**

-1

Maple is case sensitive. Pi results in the irrational number  $\pi$  (that is, 3.14159 to five decimal place); whereas pi is the 16th letter ( $\pi$ , lower case) of the Greek alphabet. PI is capital  $\pi$ ; that is:  $\Pi$ . Case sensitivity is shown by the following commands. In the next section, it will be shown how to evaluate predefined constants.

> **pi; PI; alpha; Alpha; omega; Omega;**

$\pi$

$\Pi$

$\alpha$

A

$\omega$

$\Omega$

> **Infinity; InFiNiTy; infinity;**

*Infinity*

*InFiNiTy*

$\infty$

>

**EXACT NUMBERS AND ARITHMETIC VERSUS DECIMAL APPROXIMATIONS.** Unlike calculators, Maple does exact arithmetic instead of using decimal approximations. This is illustrated by the following commands.

> **120/48;**

$$\frac{5}{2}$$

> **%-1/2;**

2

>  **%+Pi+2\*Pi;**

$$2 + 3 \pi$$

```
> %+5*exp(3);
```

$$2 + 3 \pi + 5 e^3$$

To change this to a decimal approximation, called a *floating-point number* in Maple, we use the *evalf* command, which stands for "*evaluate using floating-point arithmetic*".

```
> evalf(%); evalf(2+3*Pi+5*exp(3));
```

111.8524626

111.8524626

If we wish a decimal approximation for  $120/48$ , we use the following command:

```
> evalf(120/48);
```

2.500000000

More examples:

```
> evalf(Pi); evalf(exp(1));
```

3.141592654

2.718281828

The next command evaluates  $\pi$  to 50 decimal places:

```
> evalf(Pi,50);
```

3.1415926535897932384626433832795028841971693993751

Another way to force Maple to use decimal approximations is to use decimal places. Compare the responses to the following commands.

```
> 12/36; 12./36; 12./36.; 12.00/36.0;
```

$$\frac{1}{3}$$

0.3333333333

0.3333333333

0.3333333333

```
> 2*Pi/10; 2*Pi/10.; 10*exp(1); 10*exp(1.);
```

```
>
```

$$\frac{1}{5} \pi$$

0.2000000000  $\pi$

10 e

27.18281828

```
>
```

**THE COMMAND: RESTART.** The command **restart** should be one of the very first, and probably the very first, Maple command of every worksheet. The reason for this is that **restart** clears Maple's internal memory and allows the user to start calculations all over again without having to exit the worksheet, thereby losing the text and commands that have already been written. In other words, all past calculations and previously defined variables will be cleared from Maple's memory and will not interfere with future calculations. It is similar to writing down formulas on a whiteboard, assigning values to the variables and performing calculations, and then deciding that some of the variable assignments or calculations, or both, need to be changed. The command **restart** is like erasing all calculations and the values assigned to all of the variables from the board--but not the formulas and accompanying text--and then redoing the calculations. Let's execute the command by hitting *ENTER*.

```
>
> restart;
```

Even though it may seem as if nothing has happened after hitting *ENTER*, everything has been cleared from Maple's memory. To convince you of this, let's try the following experiment. First the letter  $x$  has been typed followed by a semicolon.

```
>
> x;
```

$x$

Note that "x" is the response. A response to a command, such as this one, is called an *output*. This particular output results in the variable called  $x$ .

```
>
```

To assign a value to a variable, we use the *assignment operator*, which consists of the colon followed by the equal sign ( $:=$ ). The command

**name := value**

assigns "value" to "name". For example, to assign "x" the value "10", we use the command:

```
> x:=10;
```

$x := 10$

Now whenever the command "x;" is executed, the response will be 10.

```
> x;
```

10

At any time, we are free to reassign the variable  $x$  another value or even an expression. Notice the outputs of the following commands.

```
> x:=p+q+5;
```

$x := p + q + 5$

```
> x;
```

$p + q + 5$

```
> x-7;
```

$p + q - 2$

Now observe what happens when we execute the command **restart** followed by  $x$ ;

```
> restart;
```

```
> x;
```

$x$

```
>
```

Note that Maple no longer remembers any of the previous values of  $x$ .

The command **restart** removes the values of all the variables. What if we only wanted to reset the value of  $x$  but nothing else? This can be done by typing:  $x := 'x'$ . Observe the output of the following commands:

```
> x:=1;
```

$x := 1$

```
> x;
```

1

```
> x:='x';
```

$x := x$

```
> x;
```

$x$

>  
**ALGEBRA.** Let's experiment with Maple's ability to handle algebra. Again before you execute each of the following commands, predict how Maple will respond.

>  $(x+y)^2$ ;  
$$(x + y)^2$$

> **expand**((x+y)^2);  
$$x^2 + 2xy + y^2$$

>  $(x+2)*(y+4)$ ;  
$$(x + 2)(y + 4)$$

> **expand**((x+2)\*(y+4));  
$$xy + 4x + 2y + 8$$

Note that Maple does not automatically multiply out the product of algebraic expressions; but the **expand** command will force Maple to carry out multiplications. Let's try another one.

>  $(3*x+2*y)^4*(x+y)$ ;  
$$(3x + 2y)^4(x + y)$$

It is cumbersome to have to retype the expression again as the argument of the **expand** command. This is where we can use the ditto operator again (%). So, instead of typing: **expand** ((3x + 2y)<sup>4</sup>(x + y)), we can merely type: **expand** (%). Let's try it.

> **expand** (%);  
$$81x^5 + 297x^4y + 432x^3y^2 + 312x^2y^3 + 112xy^4 + 16y^5$$

There is also the reverse of the **expand** command, namely, the **factor** command.

> **factor**(%);  
$$(3x + 2y)^4(x + y)$$

> **factor**(2\*x^2-x-15);  
$$(2x + 5)(x - 3)$$

>  
**BUILT-IN FUNCTIONS.** Some built-in functions along with the corresponding Maple name are the following:

- trigonometric functions: *cos*, *sin*, *tan*, *sec*, *csc*, *cot*
- inverse trigonometric functions: *arccos*, *arcsin*, *arctan*, *arcsec*, *arccsc*, *arccot*
- natural exponential function: *exp*
- natural logarithm function: *ln* or *log*
- square root function: *sqrt*

>  
**Examples.**

> **sin**(Pi/4);  
$$\frac{1}{2}\sqrt{2}$$

> **tan**(3);  
$$\tan(3)$$

> **tan**(Pi/2);  
Error, (in tan) numeric exception: division by zero

> **ln**(2); **evalf**(%); **log**(2.);  
$$\ln(2)$$
  
$$0.6931471806$$

0.6931471806

>

**USER-DEFINED FUNCTIONS.** We end this worksheet by showing how to define functions using *functional operators*, which are created with *arrow notation*: an arrow is made by typing a *minus sign* (-) followed by the *greater than sign* (>). An arrow is placed between the variable and an algebraic expression. We illustrate with some examples.

**Examples.**

> **restart:**

First we define the function  $f(x) = x + 5$  and then evaluate at various values of  $x$  and expressions.

> **f:=x->x+5;**

$$f := x \rightarrow x + 5$$

> **f(2);**

7

> **f(4); f(Pi); f(PI); f(t); f(y+h+1);**

9

$$\pi + 5$$

$$\Pi + 5$$

$$t + 5$$

$$y + h + 6$$

Now we define another function:

> **g:=s->s^2-3\*s+sin(s);**

$$g := s \rightarrow s^2 - 3s + \sin(s)$$

> **g(0); g(Pi/2);**

0

$$\frac{1}{4} \pi^2 - \frac{3}{2} \pi + 1$$

> **evalf(%);**

-1.244987880

> **f(10)+5\*g(1);**

$$5 + 5 \sin(1)$$

> **evalf(%,4);**

9.208

>

Another way in which to define functions is to use the *unapply* command, which returns a function from an algebraic expression. We illustrate with the above algebraic expressions.

> **restart:**

> **A:= x+5;**

$$A := x + 5$$

> **f:= unapply(A,x);**

$$f := x \rightarrow x + 5$$

> **f(4); f(Pi); f(PI); f(t); f(y+h+1);**

9

$$\pi + 5$$

$$\begin{aligned} &\pi + 5 \\ &t + 5 \\ &y + h + 6 \end{aligned}$$

>

Now for the second function:

> **B := s^2-3\*s+sin(s);**

$$B := s^2 - 3s + \sin(s)$$

> **g := unapply(B,s);**

$$g := s \rightarrow s^2 - 3s + \sin(s)$$

> **g(0); g(Pi/2);**

$$\begin{aligned} &0 \\ &\frac{1}{4}\pi^2 - \frac{3}{2}\pi + 1 \end{aligned}$$

> **evalf(%);**

$$-1.244987880$$

> **f(10)+5\*g(1);**

$$5 + 5 \sin(1)$$

> **evalf(%,4);**

$$9.208$$

>

>

**Exercise.**

Define the function  $f$  by  $f(x) = \frac{2\pi e^{2x}}{3\sqrt{x}} - 4 \sin\left(\frac{x}{5}\right)$ .

(a) Use the arrow notation to define the function in Maple. Evaluate  $f$  at  $x = 2.5$ .

(b) Use the unapply command to define the function in Maple. Evaluate  $f$  to 20 decimal places at  $x = \pi$ .

>

>

**\*\* End of "Getting to Know Maple" \*\***

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