Laplace Transforms

$$L\{1\} = \frac{1}{s}, \quad s > 0$$

$$L\{t^n\} = \frac{n!}{s^{n+1}}, \quad s > 0, \quad n = 0, 1, 2, 3, \ldots$$

$$L\{e^{bt}\} = \frac{1}{s-b}, \quad s > b$$

$$L\{\sin bt\} = \frac{b}{s^2 + b^2}, \quad s > 0$$

$$L\{\cos bt\} = \frac{s}{s^2 + b^2}, \quad s > 0$$

$$L\{u(t-b)\} = \frac{e^{-bs}}{s}, \quad s > 0, \quad b \geq 0$$

$$L\{\delta(t-b)\} = e^{-bs}, \quad b \geq 0$$

Theorem (Shift Property in the s-Domain). *If* $L\{f(t)\} = F(s)$ *for* $s > a$*, then*

$$L\{e^{bt} f(t)\} = F(s-b)$$

for $s > a + b$*.*

Theorem (Derivatives of the Transform Property). *Let* $f(t)$ *be a continuous function on* $[0, \infty)$ *and of exponential order* γ*. If* $L\{f(t)\} = F(s)$*, then*

$$L\{t^n f(t)\} = (-1)^n \frac{d^n}{ds^n} F(s)$$

for $s > \gamma$ *and* $n = 1, 2, 3, \ldots$*.*

Theorem (Shift Property in the t-Domain). *Let* $b \geq 0$*. If* $L\{f(t)\} = F(s)$ *for* $s > a$*, then also for* $s > a$*,*

$$L\{f(t)u(t-b)\} = e^{-bs} L\{f(t+b)\}$$

or equivalently,

$$L\{f(t-b)u(t-b)\} = e^{-bs} L\{f(t)\} = e^{-bs} F(s)$$