EXAMPLE.

$$\begin{cases} x' = 4x - 3y \\ y' = 8x - 6y \end{cases} \text{ or } \mathbf{x}' = \begin{bmatrix} 4 & -3 \\ 8 & -6 \end{bmatrix} \mathbf{x}.$$

$$\lambda^2 + 2\lambda = \lambda(\lambda - 2) = 0 \Longrightarrow \lambda = 0 \text{ and } \lambda = -2 \text{ are eigenvalues.}$$

$\boxed{\lambda = 0}$

$$(A - \lambda I)\mathbf{v} = \mathbf{0} \Longrightarrow \begin{bmatrix} 4 & -3 \\ 8 & -6 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \Longrightarrow$$

$$\begin{cases} 4v_1 - 3v_2 = 0 \\ 8v_1 - 6v_2 = 0 \end{cases} \Longrightarrow v_1 = \frac{3}{4} v_2.$$

Let $v_2 = 4 \Longrightarrow v_1 = 3$. Then

$$\mathbf{x}^1 = e^{0t} \begin{bmatrix} 3 \\ 4 \end{bmatrix} = \begin{bmatrix} 3 \\ 4 \end{bmatrix}$$

is a solution.

$\boxed{\lambda = -2}$

$$(A - \lambda I)\mathbf{v} = \mathbf{0} \Longrightarrow \left(\begin{bmatrix} 4 & -3 \\ 8 & -6 \end{bmatrix} - \begin{bmatrix} -2 & 0 \\ 0 & -2 \end{bmatrix} \right) \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \Longrightarrow$$

$$\begin{bmatrix} 6 & -3 \\ 8 & -4 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \Longrightarrow \begin{cases} 6v_1 - 3v_2 = 0 \\ 8v_1 - 4v_2 = 0 \end{cases} \Longrightarrow v_2 = 2v_1.$$

Let $v_1 = 1 \Longrightarrow v_2 = 2$. Then

$$\mathbf{x}^2 = e^{-2t} \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

is a solution. Then the general solution is

$$\mathbf{x} = c_1 \begin{bmatrix} 3 \\ 4 \end{bmatrix} + c_2 e^{-2t} \begin{bmatrix} 1 \\ 2 \end{bmatrix} \text{ or } \begin{Bmatrix} x = 3c_1 + c_2 e^{-2t} \\ y = 4c_1 + 2c_2 e^{-2t} \end{Bmatrix}.$$

MAPLE. See distinctreal.mw or distinctreal.pdf.