PROBLEM (Page 803 #30). Identify the geometric shape given by

$$x^2 + x + y^2 - y + z^2 = \frac{7}{2}.$$

Solution

Rewrite as

$$(x^2 + x \qquad) + (y^2 - y \qquad) + z^2 = \frac{7}{2}.$$

Complete squares:

$$\left(x^2 + x + \frac{1}{4}\right) + \left(y^2 - y + \frac{1}{4}\right) + z^2 = \frac{7}{2} + \frac{1}{4} + \frac{1}{4} \Longrightarrow$$

$$\left(x + \frac{1}{2}\right)^2 + \left(y - \frac{1}{2}\right)^2 + z^2 = 4$$

Thus we have the sphere of radius 2 centered at $\left(-\frac{1}{2}, \frac{1}{2}, 0\right)$.

PROBLEM (Page 802 #22). Find a vector of magnitude 3 in the same direction as $\mathbf{v} = 3\mathbf{i} + 3\mathbf{j} - \mathbf{k}$.

Solution

$$\|v\| = \sqrt{3^2 + 3^2 + (-1)^2} = \sqrt{19}$$

Thus

$$\mathbf{u} = \frac{1}{\sqrt{19}}\mathbf{v}$$

is a unit vector in the same direction of $\mathbf{v}$, so

$$3\mathbf{u} = \frac{3}{\sqrt{19}}\mathbf{v} = \frac{9}{\sqrt{19}}\mathbf{i} + \frac{9}{\sqrt{19}}\mathbf{j} - \frac{3}{\sqrt{19}}\mathbf{k}$$

has magnitude 3 in the same direction as $\mathbf{v}$.

PROBLEM (Page 803 #38). An equation for the x-axis is

$$y^2 + z^2 = 0 \quad \text{or} \quad y = z = 0.$$