PROBLEM (Page 803 #52). Three ropes are attached to a 300-pound crate. Rope A exerts a force of $\langle 10, -130, 200\rangle$ pounds on the crate, and rope B exerts a force of $\langle -20, 180, 160\rangle$ pounds on the crate. What force must rope C exert on the crate to move it up and to the right with a constant force of $\langle 0, 30, 20\rangle$ pounds.

Solution

Force of rope A is $\mathbf{a} = \langle 10, -130, 200\rangle$, rope B is $\mathbf{b} = \langle -20, 180, 160\rangle$, rope C is $\mathbf{c} = \langle c_1, c_2, c_3\rangle$, and the weight is $\mathbf{w} = \langle 0, 0, -300\rangle$.

We want the net force

$$\mathbf{a} + \mathbf{b} + \mathbf{c} + \mathbf{w} = \langle 0, 30, 20\rangle \Longrightarrow$$
$$\langle -10 + c_1, 50 + c_2, 60 + c_3\rangle = \langle 0, 30, 20\rangle \Longrightarrow$$
$$c_1 = 10, \quad c_2 = -20, \quad c_3 = -40 \quad \Longrightarrow$$
$$\mathbf{c} = \langle 10, -20, -40\rangle \Longrightarrow \|c\| \approx 45.8.$$

Thus rope C exerts a force of about 45.8 pounds in the direction $\langle 1, -2\ -4\rangle$.

MAPLE. See vectorsinspace(10.2).mw or vectorsinspace(10.2).pdf

3. The Dot Product

This is also often referred to as the scalar product.

In V_2:

$$\mathbf{a} \cdot \mathbf{b} = \langle a_1, a_2\rangle \cdot \langle b_1, b_2\rangle = a_1b_1 + a_2b_2$$

In V_3:

$$\mathbf{a} \cdot \mathbf{b} = \langle a_1, a_2, a_3\rangle \cdot \langle b_1, b_2, b_3\rangle = a_1b_1 + a_2b_2 + a_3b_3$$

EXAMPLE. If $\mathbf{a} = \langle -3, 2, 5\rangle$ and $\mathbf{b} = \langle 3, 5, -6\rangle$,

$$\mathbf{a} \cdot \mathbf{b} = \langle -3, 2, 5\rangle \cdot \langle 3, 5, -6\rangle = (-3)(3) + 2(5) + 5(-6) = -9 + 10 - 30 = -29$$