NOTE.

(1) The dot product of two vectors is a number.

(2) A vector $\langle a, b\rangle$ in V_2 can be thought of a vector $\langle a, b, 0\rangle$ in A_3, so results we prove for V_3 also hold for V_2.

THEOREM (3.1). *For vectors* $\mathbf{a}, \mathbf{b}, \mathbf{c}$ *and any scalar* d,

(1) $\mathbf{a} \cdot \mathbf{b} = \mathbf{b} \cdot \mathbf{a}$

(2) $\mathbf{a}(\mathbf{b} + \mathbf{c}) = \mathbf{a} \cdot \mathbf{b} + \mathbf{a} \cdot \mathbf{c}$

(3) $(d\mathbf{a}) \cdot \mathbf{b} = d(\mathbf{a} \cdot \mathbf{b}) = \mathbf{a} \cdot (d\mathbf{b})$

(4) $\mathbf{0} \cdot \mathbf{a} = \mathbf{0}$

(5) $\mathbf{a} \cdot \mathbf{a} = \|\mathbf{a}\|^2$

PROOF.

$$
\begin{aligned}
(2) \quad \mathbf{a} \cdot (\mathbf{b} + \mathbf{c}) &= \langle a_1, a_2, a_3\rangle \cdot \big(\langle b_1, b_2, b_3\rangle + \langle c_1, c_2, c_3\rangle\big) \\
&= \langle a_1, a_2, a_3\rangle \cdot \langle b_1 + c_1, b_2 + c_2, b_3 + c_3\rangle \\
&= a_1(b_1 + c_1) + a_2(b_2 + c_2) + a_3(b_3 + c_3) \\
&= a_1b_1 + a_1c_1 + a_2b_2 + a_2c_2 + a_3b_3 + a_3c_3 \\
&= a_1b_1 + a_2b_2 + a_3b_3 + a_1c_1 + a_2c_2 + a_3c_3 \\
&= \langle a_1, a_2, a_3\rangle \cdot \langle b_1, b_2, b_3\rangle + \langle a_1, a_2, a_3\rangle \cdot \langle c_1, c_2, c_3\rangle \\
&= \mathbf{a} \cdot \mathbf{b} + \mathbf{a} \cdot \mathbf{c}.
\end{aligned}
$$

$$(5) \quad \mathbf{a} \cdot \mathbf{a} = \langle a_1, a_2, a_3\rangle \cdot \langle a_1, a_2, a_3\rangle = a_1^2 + a_2^2 + a_3^2 = \|\langle a_1, a_2, a_3\rangle\|^2 = \|\mathbf{a}\|^2.$$

NOTE. $\mathbf{a} \cdot \mathbf{b} = 0$ does not imply $\mathbf{a} = 0$ or $\mathbf{b} = 0$, so this differs from our usual multiplication.