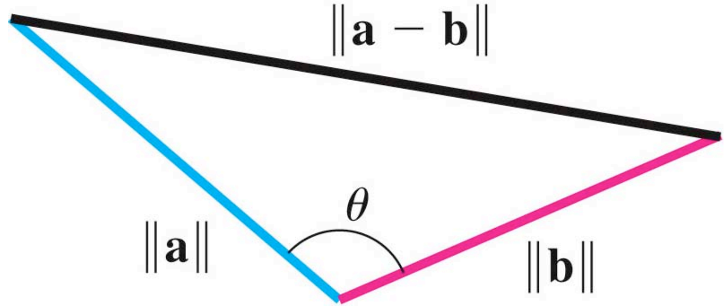
For $\mathbf{a} \neq \mathbf{0}$ and $\mathbf{b} \neq \mathbf{0}$ in V_3, the angle θ $(0 \leq \theta \leq \pi)$ between $\mathbf{a}$ and $\mathbf{b}$ is the smaller of the two angles in the plane formed by giving both vectors the same initial point.



If $\mathbf{a}$ and $\mathbf{b}$ have the same direction, $\theta = 0$.

If $\mathbf{a}$ and $\mathbf{b}$ have the opposite direction, $\theta = \pi$.

$\mathbf{a}$ and $\mathbf{b}$ are orthogonal (or perpendicular) if $\theta = \dfrac{\pi}{2}$.

We say $\mathbf{0}$ is orthogonal to every vector.

Law of Cosines

$$\|\mathbf{a} - \mathbf{b}\|^2 = \|\mathbf{a}\|^2 + \|\mathbf{b}\|^2 - 2\|\mathbf{a}\|\|\mathbf{b}\| \cos\theta$$

THEOREM (3.2). *Let θ be the angle between nonzero vectors $\mathbf{a}$ and $\mathbf{b}$. Then*

$$\mathbf{a} \cdot \mathbf{b} = \|\mathbf{a}\|\|\mathbf{b}\| \cos\theta.$$

COROLLARY. *For nonzero vectors $\mathbf{a}$ and $\mathbf{b}$,*

$$\cos\theta = \frac{\mathbf{a} \cdot \mathbf{b}}{\|\mathbf{a}\|\|\mathbf{b}\|}.$$

EXAMPLE. Find the angle between $\mathbf{a} = \langle 2, 3, 6\rangle$ and $\mathbf{b} = \langle -2, 1, 4\rangle$.

$$\cos\theta = \frac{2(-2) + 3(1) + 6(4)}{\sqrt{4+9+36}\sqrt{4+1+16}} = \frac{23}{7\sqrt{21}} \Longrightarrow$$

$$\theta = \arccos\left(\frac{23}{7\sqrt{21}}\right) \approx .7713^R \approx 44.19^\circ$$