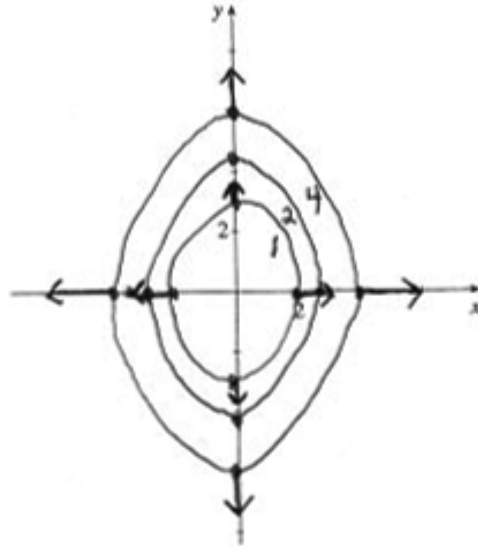


Gradient Fields and Level Curves

Compute the gradient fields for the following functions, and draw level curves $f(x, y) = k$ for the indicated values of k . Then sketch the gradient vector field at one or two points on each of these level curves.

1. $f(x, y) = \frac{x^2}{4} + \frac{y^2}{9}; k = 1, 2, 4$

$$\nabla f(x, y) = \left(\frac{x}{2}, \frac{2y}{9} \right)$$



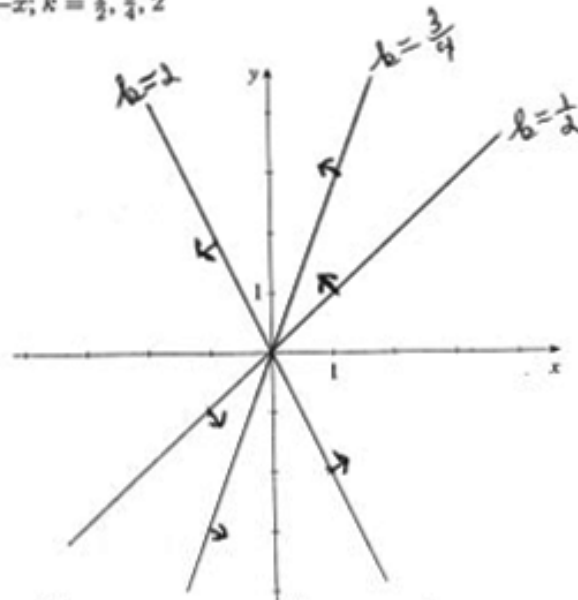
$$k=1: \frac{x^2}{4} + \frac{y^2}{9} = 1$$

$$k=2: \frac{x^2}{8} + \frac{y^2}{18} = 1$$

$$k=4: \frac{x^2}{16} + \frac{y^2}{36} = 1$$

2. $f(x, y) = \frac{y}{x+y}, y \neq -x; k = \frac{1}{2}, \frac{3}{4}, 2$

$$\nabla f(x, y) = \left(-\frac{y}{(x+y)^2}, \frac{x}{(x+y)^2} \right)$$



$$k = \frac{1}{2}: y = x$$

$$k = \frac{3}{4}: y = 3x$$

$$k = 2: y = -2x$$

$$k = 2: y = -2x$$

$$y = -2x$$

In general, the further from the origin, the shorter the vector.