

COROLLARY. \mathbf{a} and \mathbf{b} are orthogonal ($\mathbf{a} \perp \mathbf{b}$) $\iff \mathbf{a} \cdot \mathbf{b} = 0$.

THEOREM (3.3 — Cauchy-Schwarz Inequality). For any vectors \mathbf{a} and \mathbf{b} ,

$$|\mathbf{a} \cdot \mathbf{b}| \leq \|\mathbf{a}\| \|\mathbf{b}\|$$

or

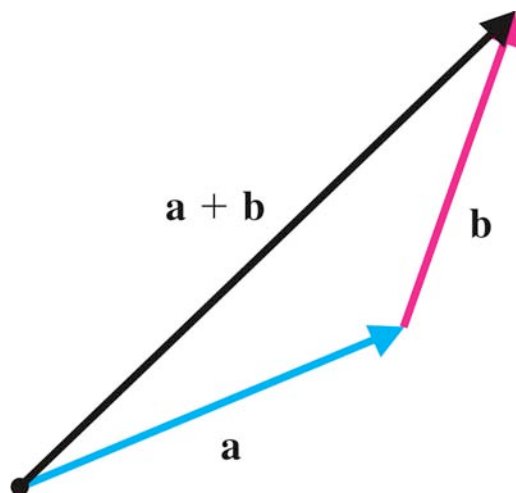
$$|a_1b_1 + a_2b_2 + a_3b_3| \leq \sqrt{a_1^2 + a_2^2 + a_3^2} \sqrt{b_1^2 + b_2^2 + b_3^2}.$$

PROOF.

$$|\mathbf{a} \cdot \mathbf{b}| = |\|\mathbf{a}\| \|\mathbf{b}\| \cos \theta| = \|\mathbf{a}\| \|\mathbf{b}\| |\cos \theta| \leq \|\mathbf{a}\| \|\mathbf{b}\|.$$

THEOREM (3.4 — Triangle Inequality). For any vectors \mathbf{a} and \mathbf{b} ,

$$\|\mathbf{a} + \mathbf{b}\| \leq \|\mathbf{a}\| + \|\mathbf{b}\|.$$



PROOF.

$$\begin{aligned} \|\mathbf{a} + \mathbf{b}\|^2 &= (\mathbf{a} + \mathbf{b}) \cdot (\mathbf{a} + \mathbf{b}) = \mathbf{a} \cdot \mathbf{a} + \mathbf{a} \cdot \mathbf{b} + \mathbf{b} \cdot \mathbf{a} + \mathbf{b} \cdot \mathbf{b} \\ &= \|\mathbf{a}\|^2 + 2\mathbf{a} \cdot \mathbf{b} + \|\mathbf{b}\|^2 \\ &\leq \|\mathbf{a}\|^2 + 2\|\mathbf{a}\| \|\mathbf{b}\| + \|\mathbf{b}\|^2 \\ &= (\|\mathbf{a}\| + \|\mathbf{b}\|)^2 \end{aligned}$$

Taking square roots then gives the result.