$$\underbrace{\oint_{C_1} M(x,y)\,dx + N(x,y)\,dy + \oint_{C_2} M(x,y)\,dx + N(x,y)\,dy}_{\text{Since the line integrals over the slits cancel}} = \oint_C M(x,y)\,dx + N(x,y)\,dy$$

where $C = C_1 \cup C_2$.

NOTE. This procedure can be extended to any finite number of holes.

EXAMPLE. Suppose $\mathbf{F}(x,y) = \left\langle \dfrac{-y}{x^2+y^2}, \dfrac{x}{x^2+y^2} \right\rangle$. Show that

$$\oint_C \mathbf{F}(x,y)\cdot d\mathbf{r} = 2\pi$$

for every simple closed curve C enclosing the origin.

SOLUTION. Green's Theorem doesn't apply since $\mathbf{F}(0,0)$ is not defined. Let C be any simple closed curve containing the origin and let C_1 be the circle of radius $a > 0$ centered at the origin, positively oriented, where a is sufficiently small so that C and C_1 do not meet. Let R be the region between and including the curves.