Applying the extended Green's Theorem,

$$\oint_C \mathbf{F}(x,y)\cdot d\mathbf{r} - \oint_{C_1} \mathbf{F}(x,y)\cdot d\mathbf{r} = \oint_{\partial R} \mathbf{F}(x,y)\cdot d\mathbf{r} = \iint_R \left( \frac{\partial N}{\partial x} - \frac{\partial M}{\partial y} \right) dA =$$

$$\iint_R \left[ \frac{(1)(x^2+y^2) - x(2x)}{(x^2+y^2)^2} - \frac{(-1)(x^2+y^2) + y(2y)}{(x^2+y^2)^2} \right] dA = \iint_R 0\, dA = 0 \Longrightarrow$$

$$\oint_C \mathbf{F}(x,y)\cdot d\mathbf{r} = \oint_{C_1} \mathbf{F}(x,y)\cdot d\mathbf{r}.$$

Parameterize $C_1$ by

$$x = a\cos t, \qquad y = a\sin t, \qquad 0 \le t \le 2\pi.$$

Then $x^2 + y^2 = a^2$, and

$$\oint_C \mathbf{F}(x,y)\cdot d\mathbf{r} = \oint_{C_1} \mathbf{F}(x,y)\cdot d\mathbf{r} = \oint_{C_1} \left\langle \frac{-y}{a^2}, \frac{x}{a^2} \right\rangle \cdot d\mathbf{r} =$$

$$\frac{1}{a^2} \oint_{C_1} (-y)\, dx + x\, dy = \frac{1}{a^2} \int_0^{2\pi} \big[ (\alpha \sin t)(-a\sin t) + (a\cos t)(a\cos t) \big]\, dt =$$

$$\int_0^{2\pi} \left( \sin^2 t + \cos^2 t \right) dt = \int_0^{2\pi} dt = t\Big|_0^{2\pi} = 2\pi.$$

□