4. The Cross Product

This is a product that exists only in 3 dimensions. We fist need an introduction to determinants for computational purposes.

DEFINITION. For a 2×2 ("2 by 2") matrix $M = \begin{bmatrix} a_1 & a_2 \\ b_1 & b_2 \end{bmatrix}$, the <u>determinant</u> of M is

$$|M| = \begin{vmatrix} a_1 & a_2 \\ b_1 & b_2 \end{vmatrix} = a_1b_2 - a_2b_1.$$

DEFINITION. The <u>determinant</u> of a 3×3 matrix M is

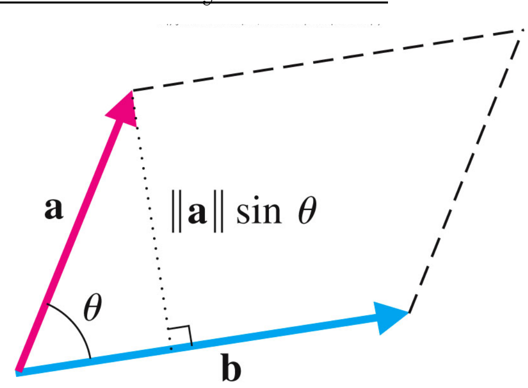
$$|M| = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} = a_1 \begin{vmatrix} b_2 & b_3 \\ c_2 & c_3 \end{vmatrix} - a_2 \begin{vmatrix} b_1 & b_3 \\ c_1 & c_3 \end{vmatrix} + a_3 \begin{vmatrix} b_1 b_2 \\ c_1 c_2 \end{vmatrix}$$

$$= a_1b_2c_3 - a_1b_3c_2 + a_2b_3c_1 - a_2b_1c_3 + a_3b_1c_2 - a_3b_2c_1.$$

EXAMPLE.

$$\begin{vmatrix} 2 & -3 & 4 \\ 5 & 1 & -1 \\ 6 & -2 & 3 \end{vmatrix} = 6 - 4 + 18 + 45 - 40 - 24 = 1$$

<u>Area of the Parallelogram formed by **a** and **b**</u>



$$\text{Area} = (\text{base}) \cdot (\text{height}) = \|\mathbf{a}\|\|\mathbf{b}\| \sin\theta$$