Planes in $\mathbb{R}^3$

A plane in space is determined by a vector $\mathbf{a} = \langle a_1, a_2, a_3 \rangle$ that is normal to the plane and a point $P_1(x_1, y_1, z_1)$ lying in the plane.

A vector is normal to a plane if it is orthogonal to every vector in the plane.

(1) Linear equation:

$$\mathbf{a} \cdot \overrightarrow{P_1P} = \langle a_1, a_2, a_3 \rangle \cdot \langle x - x_1, y - y_1, z - z_1 \rangle = \underbrace{a_1(x - x_1) + a_2(y - y_1) + a_3(z - z_1) = 0}_{linear\ equation\ for\ the\ plane}$$

Simplifying, every equation of the form

$$ax + by + cz + d = 0$$

is the equation of a plane with normal vector $\langle a, b, c \rangle \neq \mathbf{0}$.

(2) Vector equation:

$$\mathbf{a} \cdot \overrightarrow{P_1P} = 0$$

DEFINITION. Two planes are parallel when their normal vectors are parallel, and orthogonal when their normal vectors are orthogonal.