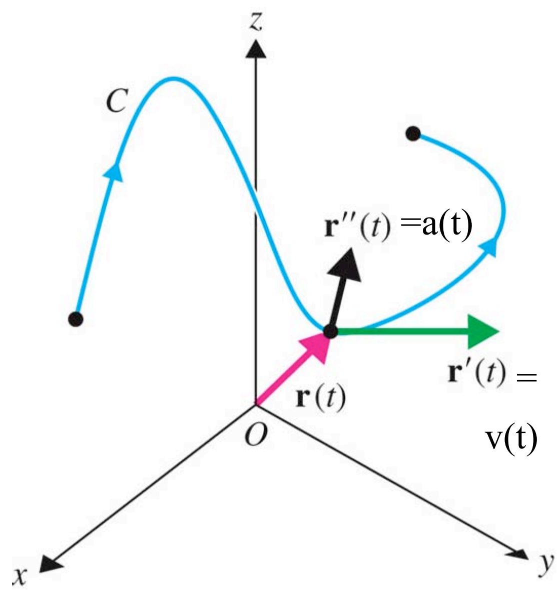
3. Motion in Space

Consider the curve traced out by $\mathbf{r}(t) = \langle f(t), g(t), h(t)\rangle$, $t \in [a, b]$.



$\mathbf{r}'(t) = \langle f'(t), g'(t), h'(t)\rangle$.

$\|\mathbf{r}'(t)\| = \sqrt{[f'(t)]^2 + [g'(t)]^2 + [h'(t)]^2}$.

For $t_0 \in [a, b]$, the arclength from $u = t_0$ to $u = t$ is

$$s(t) = \int_{t_0}^{t} \sqrt{[f'(u)]^2 + [g'(u)]^2 + [h'(u)]^2}\, du.$$

By the FTC,

$$s'(t) = \sqrt{[f'(t)]^2 + [g'(t)]^2 + [h'(t)]^2} = \|\mathbf{r}'(t)\|.$$

$s'(t)$ is the instantaneous change of arclength with respect to time = speed.

Thus $\mathbf{r}'(t)$ = velocity vector = $\mathbf{v}(t)$ and

$\mathbf{r}''(t) = v'(t)$ = accelleration vector = $\mathbf{a}(t)$.