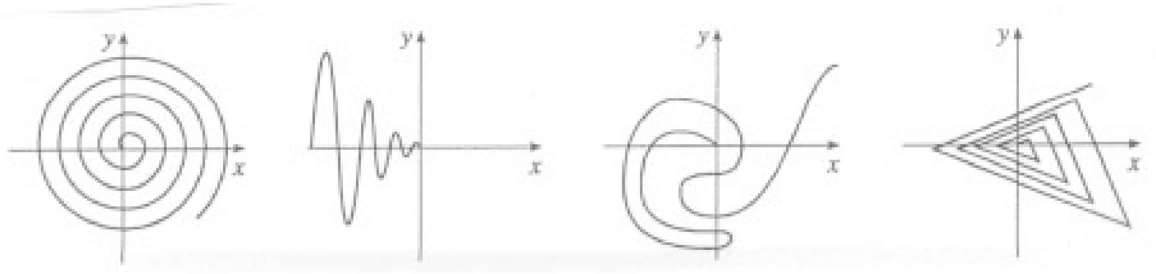
Below are some exotic paths for $(x,y)\to(0,0)$



THEOREM. *If* $\lim\limits_{(x,y)\to(a,b)} f(x,y)=L$ *and* $\lim\limits_{(x,y)\to(a,b)} g(x,y)=M$*, then*

(1) $\lim\limits_{(x,y)\to(a,b)}\big[f(x,y)\pm g(x,y)\big]=L\pm M$

(2) $\lim\limits_{(x,y)\to(a,b)}\big[f(x,y)g(x,y)\big]=LM$

(3) $\lim\limits_{(x,y)\to(a,b)}\dfrac{f(x,y)}{g(x,y)}=\dfrac{L}{M}$ *provided* $M\neq 0$.

NOTE. If a polynomial in two variables is any sum of terms of the form cx^ny^m, then the limit of the polynomial exists everywhere and is found simply by substitution.

EXAMPLE.

$$\lim_{(x,y)\to(3,2)}\frac{3xy^2+6xy}{2x^2y+4y}=\frac{3\cdot3\cdot4+6\cdot3\cdot2}{2\cdot9\cdot2+4\cdot2}=\frac{36+36}{36+8}=\frac{72}{44}=\frac{18}{11}$$

NOTE.

(1) If $f(x,y)\to L_1$ along a path P_1 toward (a,b), and $f(x,y)\to L_2\neq L_1$ along a path P_2 toward (a,b),then

$$\lim_{(x,y)\to(a,b)} f(x,y)$$

does not exist (DNE).