THEOREM (1.1). *V_2 is a vector space.*

PROOF. (of (2))

$$(\mathbf{u}+\mathbf{v})+\mathbf{w} = (\langle u_1, u_2\rangle + \langle v_1, v_2\rangle) + \langle w_1, w_2\rangle =$$
$$\langle u_1+v_1, u_2+v_2\rangle + \langle w_1, w_2\rangle = \langle (u_1+v_1)+w_1, (u_2+v_2)+w_2\rangle =$$
$$\langle u_1+(v_1+w_1), u_2+(v_2+w_2)\rangle = \langle u_1, u_2\rangle + \langle v_1+w_1, v_2+w_2\rangle =$$
$$\langle u_1, u_2\rangle + (\langle v_1, v_2\rangle + \langle w_1, w_2\rangle) = \mathbf{u}+(\mathbf{v}+\mathbf{w})$$

NOTE. For any two distinct points $A(x_1, y_1)$ and $B(x_2, y_2)$, $\overrightarrow{AB}$ corresponds to the position vector $\langle x_2 - x_1, y_2 - y_1\rangle$.

Any vector can be written in terms of the <u>standard basis vectors</u>

$$\mathbf{i} = \langle 1, 0\rangle \quad \text{and} \quad \mathbf{j} = \langle 0, 1\rangle.$$

For any $\mathbf{a} \in V_2$,

$$\mathbf{a} = \langle a_1, a_2\rangle = \langle a_1, 0\rangle + \langle 0, a_2\rangle = a_1\langle 1, 0\rangle + a_2\langle 0, 1\rangle = a_1\mathbf{i} + a_2\mathbf{j}.$$

We call a_1 and a_2 the <u>horizontal and vertical components</u> of $\mathbf{a}$

NOTE. $\|\mathbf{i}\| = \|\mathbf{j}\| = 1$.

A <u>unit vector</u> is a vector $\mathbf{a}$ where $\|\mathbf{a}\| = 1$.

THEOREM (1.2). *For any nonzero vector $\mathbf{a}$, a unit vector having the same direction as $\mathbf{a}$ is*

$$\mathbf{u} = \frac{1}{\|\mathbf{a}\|}\mathbf{a}.$$

PROOF.

$\mathbf{a} \neq \mathbf{0} \Longrightarrow \|\mathbf{a}\| > 0 \Longrightarrow \mathbf{u}$ is a positive scalar multiple of $\mathbf{a} \Longrightarrow$

$\mathbf{u}$ has the same direction as $\mathbf{a}$.

$$\|\mathbf{u}\| = \left\|\frac{1}{\|\mathbf{a}\|}\mathbf{a}\right\| = \left|\frac{1}{\|\mathbf{a}\|}\right| \cdot \|\mathbf{a}\| = \frac{1}{\|\mathbf{a}\|} \cdot \|\mathbf{a}\| = 1.$$