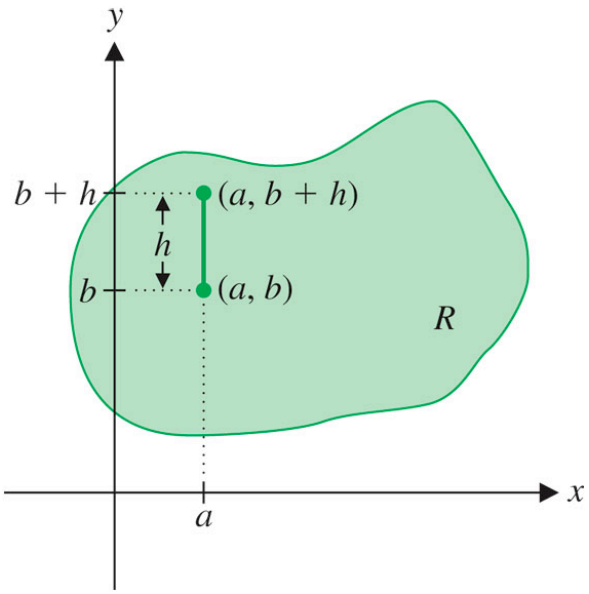
The average rate of change as you move vertically from (a, b) to $(a, b+h)$ is

$$\frac{f(a, b+h) - f(a, b)}{h}.$$



The instantaneous rate of change in the y-direction at (a, b) is

$$\frac{\partial f}{\partial y}(a, b) = \lim_{h \to 0} \frac{f(a, b+h) - f(a, b)}{h},$$

the partial derivative of f with respect to y.