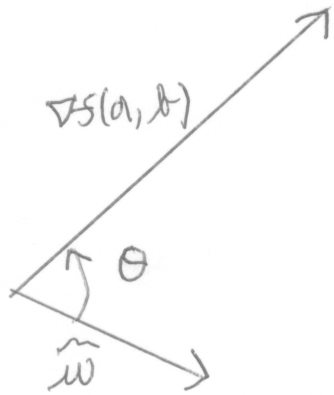
Properties of the gradient vector.

Suppose θ is the angle between $\nabla f(a,b)$ and $\mathbf{u}$.



Then

$$D_{\mathbf{u}}f(a,b) = \nabla f(a,b)\cdot\mathbf{u} = \|\nabla f(a,b)\|\cdot\|\mathbf{u}\|\cos\theta = \|\nabla f(a,b)\|\cdot\cos\theta.$$

Note that $D_{\mathbf{u}}f(a,b)$ is maximized when $\theta = 0$ or $\cos\theta = 1$. Also, $\theta = 0 \Longrightarrow \nabla f(a,b)$ and $\mathbf{u}$ are in the same direction $\Longrightarrow \mathbf{u} = \dfrac{\nabla f(a,b)}{\|\nabla f(a,b)\|}$.

Further, $D_{\mathbf{u}}f(a,b)$ is minimized when $\theta = \pi$ or $\cos\theta = -1$, i.e., $D_{\mathbf{u}}f(a,b)$ and $\mathbf{u}$ have opposite directions $\Longrightarrow \mathbf{u} = -\dfrac{\nabla f(a,b)}{\|\nabla f(a,b)\|}$.

Finally, $\theta = \dfrac{\pi}{2} \Longrightarrow \cos\theta = 0 \Longrightarrow D_{\mathbf{u}}f(a,b) = 0 \Longrightarrow \nabla f(a,b) \perp \mathbf{u} \Longrightarrow \mathbf{u}$ is tangent to the level curve at (a,b) since f is constant on level curves and $\nabla f(a,b)$ is orthogonal to the level curve at (a,b).