4. Errors in Scientific Computation

Sequencing of operations can make a difference:

PROBLEM (p.27, #1b). Use four digit rounding to find the roots of $\frac{1}{3}x^2 + \frac{123}{4}x - \frac{1}{6} = 0$. Note that the exact roots round to -92.2554197358 and $-.005419735789$.

The quadratic formula for $ax^2 + bx + c = 0$ gives

$$x_1 = \frac{-b + \sqrt{b^2 - 4ac}}{2a} \text{ and } x_2 = \frac{-b - \sqrt{b^2 - 4ac}}{2a}$$

as solutions.

$a = .3333, b = 30.75, c = -.1667, b^2 = 945.6, ac = -.0556, 4ac = -.2224$

$b^2 - 4ac = 945.8, \sqrt{b^2 - 4ac} = 30.75, -b + \sqrt{b^2 - 4ac} = 0$

$-b - \sqrt{b^2 - 4ac} = -61.50, \text{and } 2a = .6666$, so

$x_1 = \dfrac{0}{.6666} = 0$ and $x_2 = \dfrac{-61.50}{.6666} = -92.26$, and

relative error=1 and relative error=4.96×10^{-5}.

Thus we have no and 5 significant digits. But rationalizing numerators,

$$x_1 = \frac{-2c}{b + \sqrt{b^2 - 4ac}} \text{ and } x_2 = \frac{-2c}{b - \sqrt{b^2 - 4ac}},$$

so $2c = -.3334$, $b + \sqrt{b^2 - 4ac} = 61.5$, and $b - \sqrt{b^2 - 4ac} = 0$, giving

$x_1 = \dfrac{-.3334}{61.5} = .005421$ and $x_2 = \dfrac{-.3334}{0}$, undefined.

Relative error for x_1 is 2.3×10^{-4}, giving 4 significant digits. □

MAPLE. See nested.mw and/or nested.pdf