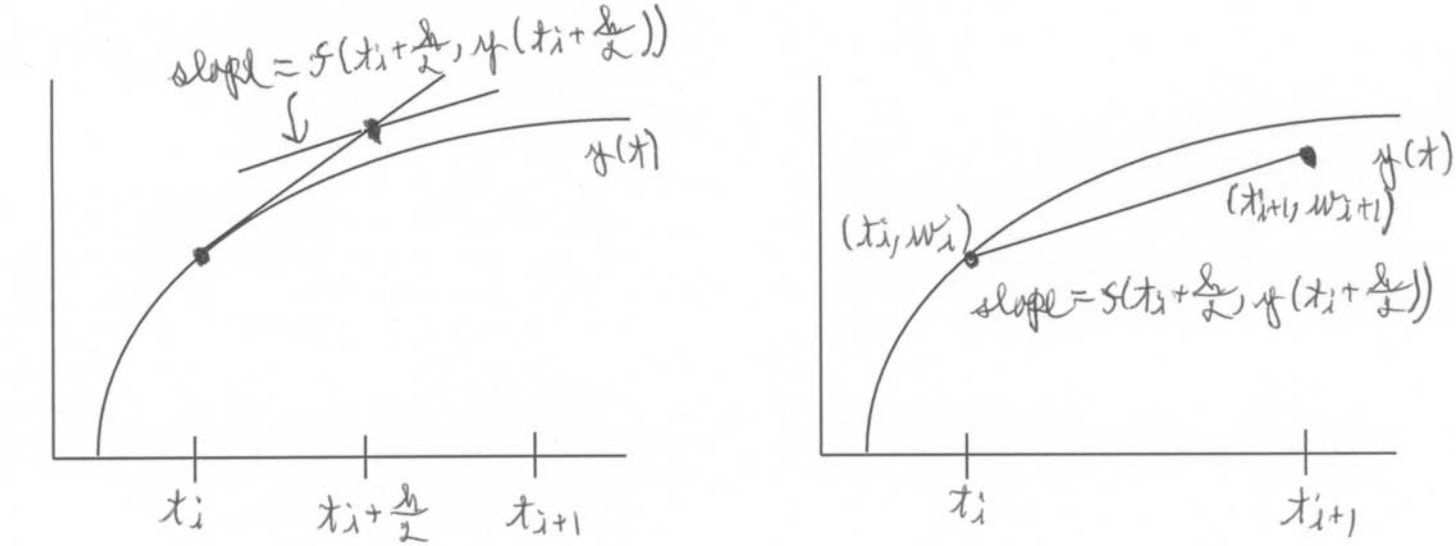

In the above figure, we see that the Midpoint Method applies the slope found at the midpoint of the interval (on the left) to the entire interval (on the right).

Using $T^{(3)}(t,y) \approx a_1 f(t,y) + a_2 f(t+\alpha, y+\beta f(t,y))$, the extra parameter gives an infinite number of second-order R–K formulas with local truncation error at best $O(h^3)$.

$a_1 = a_2 = \frac{1}{2}$ and $\alpha = \beta = h$ gives the

Modified Euler Method (classical[heunform] or submethod= meuler in Maple)

$$\begin{cases} w_0 = \alpha \\ w_{i+1} = w_i + \dfrac{h}{2}\Big[f(t_i, w_i) + f(t_{i+1}, w_i + hf(t_i, w_i))\Big] \end{cases}$$

where $i = 0, 1, \ldots, N-1$, with local error $O(h^3)$ and global error $O(h^2)$.

or

$$\begin{cases} w_0 = \alpha \\ w_p = w_i + hf(t_i, w_i) \\ w_{i+1} = w_i + \dfrac{h}{2}\Big[f(t_i, w_i) + f(t_{i+1}, w_p)\Big] \end{cases}$$

where $i = 0, 1, \ldots, N-1$, with local error $O(h^3)$ and global error $O(h^2)$.