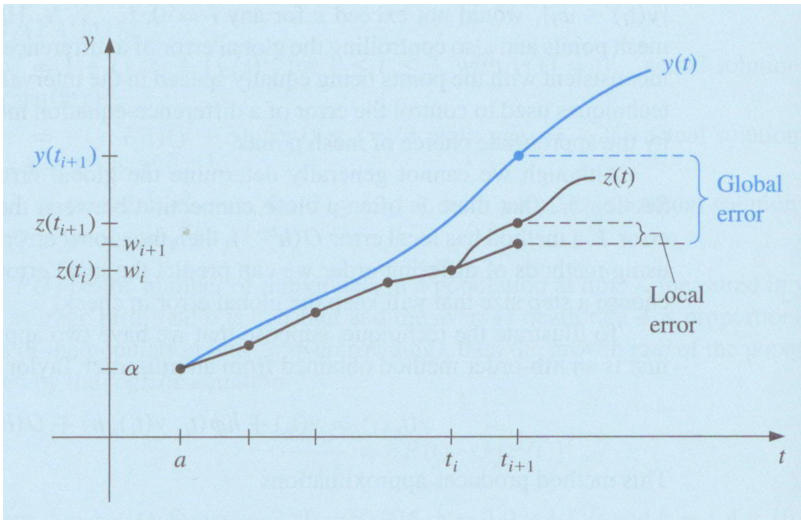
$$\begin{cases} \widetilde{w}_0 = \alpha \\ \widetilde{w}_{i+1} = \widetilde{w}_i + h\widetilde{\phi}(t_i, \widetilde{w}_i, h) \text{ for } i > 0 \end{cases}$$

with truncation error $O(h^{n+2})$ where for some $\widetilde{K}$ and all relevant h and i,

$$|y(t_i) - \widetilde{w}_i| < \widetilde{K}h^{n+1}.$$

Suppose at t_i we have $w_i = \widetilde{w}_i = z(t_i)$ where $z(t)$ is a solution satisfying $z(t_i) = w_i$, not $z(a) = \alpha$.



We now have 2 approximations w_{i+1} and $\widetilde{w}_{i+1}$ whose differences from $y(t_i + h)$ are global errors and whose differences from $z(t_i + h)$ are local errors. We have

$$\underbrace{z(t_i + h) - w_{i+1}}_{\text{local error of } O(h^{n+1})} = (\widetilde{w}_{i+1} - w_{i+1}) + (\underbrace{z(t_i + h) - \widetilde{w}_{i+1}}_{\text{local error of } O(h^{n+2})}).$$

Thus

$$z(t_i + h) - w_{i+1} \approx \widetilde{w}_{i+1} - w_{i+1} = O(h^{n+1}),$$

so a constant K exists such that

$$Kh^{n+1} = |z(t_i + h) - w_{i+1}| \approx |\widetilde{w}_{i+1} - w_{i+1}| \Rightarrow$$

$$(\star) \quad K \approx \frac{|\widetilde{w}_{i+1} - w_{i+1}|}{h^{n+1}}$$

Then find a new step size qh such that

$$|y(t_i + qh) - w_{i+1} \text{ (with step size } qh)| < K(qh)^n < \epsilon$$