The system of linear equations

$$\begin{matrix} a_{11}x_1 + \cdots + a_{1n}x_n = b_1 \\ \vdots \quad \ddots \quad \vdots \quad \vdots \\ a_{n1}x_1 + \cdots + a_{nn}x_n = b_n \end{matrix}$$

corresponds to

$$\begin{bmatrix} a_{11} & \cdots & a_{1n} \\ \vdots & \ddots & \vdots \\ a_{n1} & \cdots & a_{nn} \end{bmatrix} \begin{bmatrix} x_1 \\ \vdots \\ x_n \end{bmatrix} = \begin{bmatrix} b_1 \\ \vdots \\ b_n \end{bmatrix} \text{ or } A\mathbf{x} = \mathbf{b}.$$

$A\mathbf{x} = \mathbf{b}$ has a unique solution $\iff A$ is invertible. In that case,

$$A^{-1}(A\mathbf{x}) = A^{-1}\mathbf{b} \Longrightarrow (A^{-1}A)\mathbf{x} = A^{-1}\mathbf{b} \Longrightarrow$$

$$I_n\mathbf{x} = A^{-1}\mathbf{b} \Longrightarrow \mathbf{x} = A^{-1}\mathbf{b}$$

PROBLEM (Page 268 #2b(ii)). Find the inverse of $A = \begin{bmatrix} 1 & 2 & 0 \\ 2 & 1 & -1 \\ 3 & 1 & 1 \end{bmatrix}$, i.e., find $B$ such that

$$\begin{bmatrix} 1 & 2 & 0 \\ 2 & 1 & -1 \\ 3 & 1 & 1 \end{bmatrix} \begin{bmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} = \end{bmatrix} =$$

$$\begin{bmatrix} b_{11} + 2b_{21} & b_{12} + 2b_{22} & b_{13} + 2b_{23} \\ 2b_{11} + b_{21} - b_{31} & 2b_{12} + b_{22} - b_{32} & 2b_{13} + b_{23} - b_{33} \\ 3b_{11} + b_{21} + b_{31} & 3b_{12} + b_{22} + b_{32} & 3b_{13} + b_{23} + b_{33} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$

Matching up columns, we have 3 systems of linear equations, all with the same coefficient matrix. As a result, we can use Gaussian elimination on a larger augmented matrix to solve all 3 systems at once: