In $\mathbb{R}^1$, $\|\mathbf{x}\|_2 = \|\mathbf{x}\|_\infty = |x|$.

EXAMPLE.

$$\mathbf{x} = \begin{bmatrix} 3 \\ 4 \\ -2 \end{bmatrix}$$

$$\|\mathbf{x}\|_2 = \sqrt{3^2 + 4^2 + (-2)^2} = \sqrt{29}$$

$$\|\mathbf{x}\|_\infty = \max\left\{|3|, |4|, |-2|\right\} = 4$$

Since the norm of a vector gives "its distance from $\mathbf{0}$," the <u>distance between two vectors</u> is the norm of their difference.

If $\mathbf{x} = \begin{bmatrix} x_1 \\ \vdots \\ x_n \end{bmatrix}$ and $\mathbf{y} = \begin{bmatrix} y_1 \\ \vdots \\ y_n \end{bmatrix}$,

the l_2 distance between $\mathbf{x}$ and $\mathbf{y}$ is

$$\|\mathbf{x} - \mathbf{y}\|_2 = \left(\sum_{i=1}^{n} (x_i - y_i)^2\right)^{\frac{1}{2}}$$

and the l_∞ distance between $\mathbf{x}$ and $\mathbf{y}$ is

$$\|\mathbf{x} - \mathbf{y}\|_\infty = \max_{1 \le i \le n} |x_i - y_i|$$

EXAMPLE.

$$\mathbf{x} = \begin{bmatrix} 3 \\ 4 \\ -2 \end{bmatrix} \text{ and } \mathbf{y} = \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}$$

$$\|\mathbf{x} - \mathbf{y}\|_2 = \left\|\begin{bmatrix} 2 \\ 5 \\ -2 \end{bmatrix}\right\|_2 = \sqrt{2^2 + 5^2 + (-2)^2} = \sqrt{33}$$

$$\|\mathbf{x} - \mathbf{y}\|_\infty = \max\left\{|2|, |5|, |-2|\right\} = 5$$