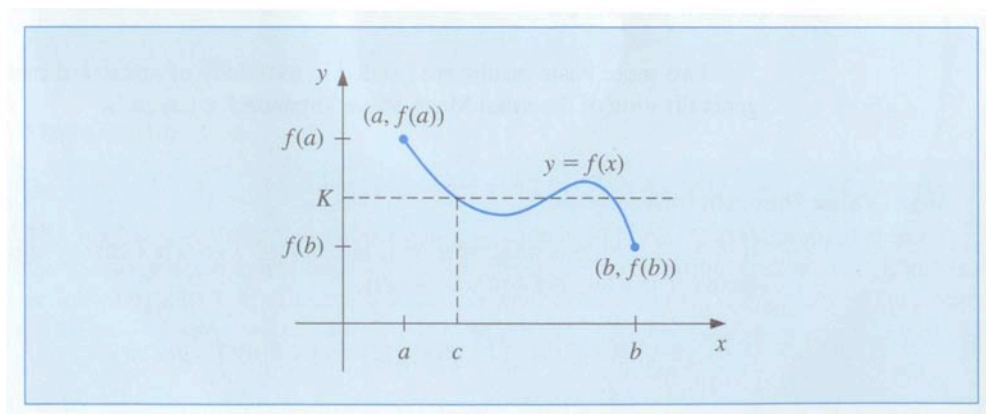


## Solutions of Equations of One Variable

**THEOREM** (Intermediate Value theorem (IVT)). *If  $f \in C[a, b]$  and  $K$  is any number between  $f(a)$  and  $f(b)$ , then there exists  $c \in (a, b)$  such that  $f(c) = K$ .*



In particular, if  $f(a)f(b) < 0$ , then there exists  $c$  in  $(a, b)$  such that  $f(c) = 0$ . Thus, one can use the IVT as a root locator.

## 2. The Bisection Method

**Problem:** find the roots of  $f(x) = 0$  where  $f$  is continuous on an open interval containing the roots.

**Solution:** first graph to locate roots by IVT. For each root  $p$ , find a closed interval  $[a, b]$  where  $p \in (a, b)$  and  $f(a) \cdot f(b) < 0$ . Make sure these intervals do not overlap.

**NOTE.** It is often better to use  $\text{sgn}(f(a)) \cdot \text{sgn}(f(b)) < 0$  rather than  $f(a) \cdot f(b) < 0$ .

$$\text{sgn}(x) = \begin{cases} -1 & x < 0 \\ 0 & x = 0 \\ 1 & x > 0 \end{cases}$$