

Then Jacobi's iterative technique becomes

$$\mathbf{x}^{(k)} = D^{-1}(L + U)\mathbf{x}^{(k-1)} + D^{-1}\mathbf{b}.$$

Letting  $T_j = D^{-1}(L + U)$  and  $\mathbf{c}_j = D^{-1}\mathbf{b}$ , we have

$$(**) \quad \mathbf{x}^{(k)} = T_j\mathbf{x}^{(k-1)} + \mathbf{c}_j.$$

In general, (\*) is used for computation and (\*\*) for theoretical purposes.

An algorithm in pseudocode for implementing Jacobi's iterative technique follows.

### Jacobi Iterative

To solve  $A\mathbf{x} = \mathbf{b}$  given an initial approximation  $\mathbf{x}^{(0)}$ :

**INPUT** the number of equations and unknowns  $n$ ; the entries  $a_{ij}$ ,  $1 \leq i, j \leq n$  of the matrix  $A$ ; the entries  $b_i$ ,  $1 \leq i \leq n$  of  $\mathbf{b}$ ; the entries  $XO_i$ ,  $1 \leq i \leq n$  of  $\mathbf{XO} = \mathbf{x}^{(0)}$ ; tolerance  $TOL$ ; maximum number of iterations  $N$ .

**OUTPUT** the approximate solution  $x_1, \dots, x_n$  or a message that the number of iterations was exceeded.

**Step 1** Set  $k = 1$ .

**Step 2** While ( $k \leq N$ ) do Steps 3–6.

**Step 3** For  $i = 1, \dots, n$

$$\text{set } x_i = \frac{-\sum_{\substack{j=1 \\ j \neq i}}^n (a_{ij} XO_j) + b_i}{a_{ii}}.$$

**Step 4** If  $\|\mathbf{x} - \mathbf{XO}\| < TOL$  then **OUTPUT** ( $x_1, \dots, x_n$ );  
(The procedure was successful.)  
**STOP**.

**Step 5** Set  $k = k + 1$ .

**Step 6** For  $i = 1, \dots, n$  set  $XO_i = x_i$ .

**Step 7** **OUTPUT** ('Maximum number of iterations exceeded');  
(The procedure was successful.)  
**STOP**. ■