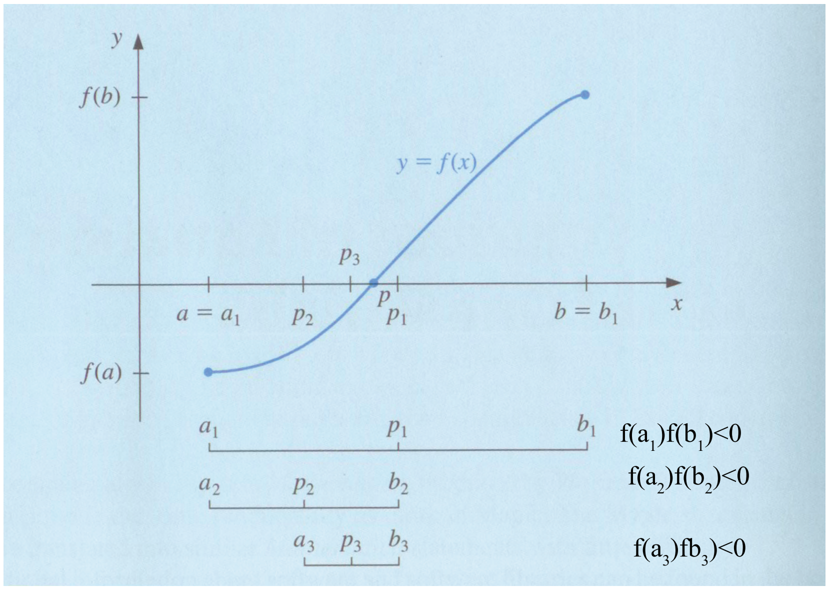
Graphical idea:



$$\{p_n\} \to p \text{ or } \lim_{n\to\infty} p_n = p$$

Method:

Given $p \in (a_i, b_i)$ with $f(a_i) \cdot f(b_i) < 0$, let

$$p_i = a_i + \frac{b_i - a_i}{2}, \text{ the midpoint of } (a_i, b_i).$$

If $f(p_i) = 0$, we are done and $p = p_i$.

If $f(p_i) \neq 0$, set $a_{i+1} = a_i$ and $b_{i+1} = p_i$ if $f(a_i) \cdot f(p_i) < 0$ and

$a_{i+1} = p_i$ and $b_{i+1} = b_i$ otherwise.

Note the use of $p_i = a_i + \dfrac{b_i - a_i}{2}$, a definite correction, instead of $p_i = \dfrac{a_i + b_i}{2}$. In the latter case, if $b_i - a_i$ is near the maximum precision of the machine, it is possible that $p_i \notin [a_i, b_i]$.

Note also that at each step $|p_n - p| \leq \dfrac{b-a}{2^n}$.