Example. Let $f(x) = 2x\cos(2x) - (x-2)^2$, $x_0 = 0$.

(a)Find $P_3(x)$ and $P_4(x)$ and use them to approximate $f(0.4)$.

$$\begin{aligned}
f'(x) &= 2\cos(2x) - 4x\sin(2x) - 2(x-2) \\
f''(x) &= -4\sin(2x) - 4\sin(2x) - 8x\cos(2x) - 2 \\
&= -8sin(2x) - 8x\cos(2x) - 2 \\
f'''(x) &= -16\cos(2x) - 8\cos(2x) + 16x\sin(2x) \\
&= -24\cos(2x) + 16x\sin(2x) \\
f^{(4)}(x) &= 48\sin(2x) + 16\sin(2x) + 32x\cos(2x) \\
&= 64\sin(2x) + 32x\cos(2x) \\
f^{(5)}(x) &= 128\cos(2x) + 32\cos(2x) - 64x\sin(2x) \\
&= 160\cos(2x) - 64x\sin(2x)
\end{aligned}$$

$$f(0) = -4; f'(0) = 6; f''(0) = -2; f'''(0) = -24; f^{(4)}(0) = 0$$

$$P_3(x) = -4 + 6x + \left(\frac{-2}{2}\right)x^2 + \left(\frac{-24}{6}\right)x^3 = -4x^3 - x^2 + 6x - 4$$

$$f(0.4) \approx P_3(0.4) = -2.016$$

Since $f^{(4)}(0) = 0$,

$$P_4(x) = P_3(x) = -4x^3 - x^2 + 6x - 4.$$