EXAMPLE. $x_0 = (2,4)$, $x_1 = (3,2)$, $x_2 = (4,6)$, so $n = 2$.

$$L_{2,0}(x) = \frac{(x-3)(x-4)}{(2-3)(2-4)} = \frac{1}{2}(x^2 - 7x + 12)$$

$$L_{2,1}(x) = \frac{(x-2)(x-4)}{(3-2)(3-4)} = -(x^2 - 6x + 8)$$

$$L_{2,2}(x) = \frac{(x-2)(x-3)}{(4-2)(4-3)} = \frac{1}{2}(x^2 - 5x + 6)$$

$$P(x) = 4\underbrace{\left(\frac{1}{2}\right)(x^2 - 7x + 12)}_{basis function} + 2\underbrace{(-1)(x^2 - 6x + 8)}_{basis function} + 6\underbrace{\left(\frac{1}{2}\right)(x^2 - 5x + 6)}_{basis function} \Longrightarrow$$

$$P(x) = 3x^2 - 17x + 26.$$

Note that each of our 3 given points are solutions of $P(x) = 0$. Also, the basis functions do not depend on the function values at our points. They are something like **i** and **j** in Physics or complex math. $\square$

THEOREM. *Suppose $x_0, x_1, \ldots, x_n$ are distinct points in $[a,b]$ and $f \in C^{n+1}[a,b]$. Then for every $x \in [a,b]$ there exists $\xi(x) \in (a,b)$ such that*

$$f(x) = \underbrace{P(x)}_{interpolating\ polynomial} + \underbrace{\frac{f^{n+1}(\xi(x))}{(n+1)!}(x-x_0)(x-x_1)\cdots(x-x_n)}_{error}.$$

NOTE. Lagrange polynomials are used in numerical differentiation and integral methods, and error bounds there depend on Lagrange error bounds.