(b) Use the error formula in Taylor's Theorem to find an upper bound for the error $|f(0.4) - P_3(0.4)|$ and $|f(0.4) - P_4(0.4)|$.

By graphing, for $0 \leq x \leq 0.4$, $|f^{(4)}(x)| \leq 55.1$ and $|f^{(5)}(x)| \leq 160$.

Then

$$|R_3(.4)| = |f(0.4) - P_3(0.4)| = \frac{|f^{(4)}(\xi(x))|}{4!}|.4 - 0|^4 \leq \frac{55.1}{24}(.4)^4 \leq .05878$$

by rounding up. Similarly,

$$|R_4(.4)| = |f(0.4) - P_4(0.4)| = \frac{|f^{(5)}(\xi(x))|}{5!}|.4 - 0|^5 \leq \frac{160}{120}(.4)^5 \leq .01366,$$

a better bound than for $|R_3(.4)|$.

The actual absolute error is

$$|f(0.4) - P_3(0.4)| = |(.8\cos(.8) - 2.56) - (-2.016)| \approx .01337.$$

Notice that in this problem we are dealing with error at a point.□